## A Joint Model of Particles and Spacetime

### Hans-Dieter Herrmann

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#### Abstract

The emergent character of space-time is a main topic in different approaches to Quantum Gravity. This article connects the emergence of space-time and the emergence of particles, capable to translate in space-time.

A model is proposed which describes the structure and dynamics of fundamental particles as well as the properties of space-time, emerging due to the structural development and the motion of particles. Particles are assumed to be composed and spatially extended in a circular extra space, called basic space, however fundamental and nearly point-like in space-time. The connected development of particles and space-time is realized in two stages.

During the one-particle stage, intrinsic particle properties such as invariant mass, charge, angular momentum (spin) and magnetic dipole moment are formed by a force-free circular motion of masses and charges. An isolated piece of space-time emerges as the translational state space of a particle, where a local mode and a nonlocal mode of translation appear. The deterministic circular motion is accompanied by stochastic linear steps, both performed with the velocity of light. The probabilistic description of intermittent translation leads to the laws of the Special Relativity Theory. During the second, multi-particle stage, the particle structure shows capabilities of interaction, and space-time develops from isolated pieces to a general space-time spanned by spatial distances between different particles. The relativistic addition of velocities and the special Lorentz transformation follow naturally from the probabilistic expressions.

In the nonlocal mode of translation, the particle model has six dimensions (D = 5 + 1). Two subcomponents of a particle, called rotons, have D = 3 + 1 dimensions and different inobservable positions in space-time. The nonlocal mode of translation corresponds to a superposion of quantum states (quantum nonlocality) and represents possibly the origin of nonlocal Newtonian gravity.

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## 1 Introduction

The recent research in Quantum Gravity shows the tendency to become a 'non-spatiotemporal theory' [16],[18],[28]. However, no accepted solution exists for the problem, how to generate space-time properties 'Out of Nowhere'. This article proposes a connection between the emergence of space-time and the emergence of particles, capable to translate in space-time.

The analogies between different levels of reality, which result in the assumption of a circular 'extra space' complementing space-time, represent the philosophical foundation of this proposal [12].

Within the Standard Model of particle physics, leptons and quarks are nearly "point-like" according to the resolution of recent scattering experiments. These fermions are considered to be "elementary particles", not structured or composed. The masses of leptons and quarks and the mixing of quarks in real particles are not explained by the Standard Model (SM). The so called 'intrinsic' properties, such as spin and magnetic dipole moment of a particle, are successfully described by the Dirac theory and by Quantum Electro-Dynamics (QED), however one needs abstract Hilbert spaces and a complex system of virtual particles in these theories. In addition, the interaction of fundamental particles and their self-interaction can be described only with certain complications because the interaction strengths and energies tend to become infinite if the distances approach zero. These facts and some other fundamental problems (such as the unknown nature of dark matter and dark energy) led to the view that the Standard Model is presumably incomplete, despite the description of the subatomic world by the SM, successful within its framework.

Since more than 100 years the main goal of newly developed particle models is to explain the circular properties such as spin and magnetic dipole moment by a circular physical motion. The motion of the electron *bound* within the hydrogen atom was described by circles (Bohr 1913[4]) and ellipses (Sommerfeld 1916 [26]), finally by standing waves (Schrödinger, 1926 [24]). These periodic motions of the electron were performed radiationless, without any loss of energy.

In a similar way a periodic motion was postulated also for the *free* electron. Slater [25] was 1926 the first to estimate a radius of circulating electromagnetic fields, matching the Compton wave-length of the electron divided by  $2\pi$ . Huang [15] expressed 1952 very clear the proposed dimension of the circulation:

"The detailed motion of a free Dirac electron is investigated .... It is shown that the well-known zitterbewegung may be looked upon as a circular motion about the direction of the electron spin, with a radius equal to the Compton wavelength (divided by  $2\pi$ ) of the electron. It is further shown that the intrinsic spin of the electron may be looked upon as the "orbital angular momentum" of this motion. The current produced by the zitterbewegung is seen to give rise to the intrinsic magnetic moment of the electron".

The Compton radius is the characteristic extension of the most ring-like or helical models, it amounts for the electron to  $\sim 386$  fm, the Compton wavelength equals  $\sim 2426$  fm.

Such models were in severe contradiction to empirical facts. Scattering experiments with electrons demonstrate their point-like character down to the region of  $\sim 10^{-3}$  fm, more than five orders of magnitude below the Compton radius.

Despite the contra-indication by experimental results, geometric models have been proposed also during the last decades [8], [22]. Hestenes developed a coordinate-free 'space-time algebra' in order to achieve a geometric interpretation of the Dirac-equation without a visible contradiction to experiments.

However, he could not avoid the contact with quantitative results in defining a 'particle clock' [14]. The time unit of this clock  $\tau_e = 4.0466 \times 10^{-21}$  s corresponds again to the rotation time needed at a circle which has a circumference of one half the Compton-wavelength of the electron, identical with Slater's result mentioned above [25]. One century of efforts to resolve the difference between a plausible geometry and the measured size of an electron had no success.

Beck [3] postulated therefore 2023 an 'internal space' for the circulating charge, but finally the radius of the 'total motion' of the electron defined by Beck equals the Compton wavelength divided by  $4\pi$ .

Models with an undetectable small extension seem to provide a solution to the problem. Such models assume an internal structure and sometimes compositeness of leptons and quarks, mostly in higher dimensions than the four-dimensional space-time. An important property of such models consists in the invisibility to experiments caused by smallness. The extremely small dimensions cannot be resolved by available accelerator energies. Therefore, the particles appear point-like in measurements while being extended.

Theodor Kaluza [17] and Oskar Klein [19] proposed already 1921 and 1926 a fifth dimension as a closed ring with a radius in the region of  $10^{-32}m$ . The possibility of compositeness of leptons and quarks was investigated for a long time [1], [20]. However, no sign of compositeness could be found experimentally [2].

Subcomponents of leptons and quarks ("preons" or "rishons") have been proposed with a small radius, not detectable experimetally. Consequently, their mass has to be very large. The appropriate minimum preon mass amounts to > 200 GeV corresponding to the inverse radius of  $\sim 10^{-18}$  m. The main difficulty of preon models is to explain, which kind of binding mechanism leads to the small masses of leptons and quarks, which are negligible in comparison to the preon mass. Therefore a coupling between preons has to be assumed, which is basically different from other couplings — such as the coupling of quarks in hadrons or the coupling of nucleons in nuclei, where always the mass of constituents is smaller than the mass of the composed systems.

Superstring theories represent also an attempt to overcome the problems with point-like particles: objects (strings with dimension 1 or branes with dimension 2) with vibrations in 6, 10 or 26 dimensions are assumed. To be compatible with the four dimensions of space-time, the extra dimensions are assumed to be compactified. Superstrings and branes are extended, however undetectably small in the region of the Planck length. Therefore, they would appear as point-like particles in all experiments. The superstring theory offered hope to many theorists to find a unified description of all particles and interactions, including gravity [11]. Unfortunately, no string theory predictions could be verified experimentally. Moreover, the landscape of possible solutions of string theory is so wide that it is extremely difficult (if not impossible) to select the solution appropriate for our universe. A severe controversy regarding the future prospects or the failure of string theory lasts over years [10].

In mathematical physics, one can define two different coordinate systems, one represents space-time and the second is a mathematical defined space inaccessible to experiments.

Hilbert spaces can be defined to describe space-time properties in this way [6], [5]. However, the character as function spaces and the big, frequently unlimited number of dimensions make of Hilbert spaces less usable for the construction of geometric particle models.

In 1979, DiVecchia and Ravndal described a supersymmetric Dirac particle [9], [23]. The theory of the particle considers its dual existence: it exists in Minkowski's space-time, as well as in an anti-commutative space spanned by Grassmann variables. The spin operator produced the correct spin of the particle only in this anti-commutative space, however, this space is not defined as a circular space. Particles with a local (internal) supersymmetry have also been studied in an external field. The theory is restricted to fields that do not disturb the supersymmetry.

Wang et al. [27] differentiate between two spaces of a Dirac particle:

"The internal degrees of freedom of a Dirac particle are related to its spinor structure called internal space  $H_c$ . The external degrees of freedom of a Dirac particle are associated with its position and momentum in space called external space  $H_p$ . The Dirac dynamics couples the internal and external space interestingly."

The disappearance of space-time as carrier of fundamental physical properties is one of the main topics in Quantum Gravity (QG). Different theories provide several interpretations of the emergent character of space-time. The approaches 'Causal Set Theory' and 'Loop Quantum Gravity' discuss space-time without a connection to its material content, particles or macroscopic objects. The 'String Theory' and the 'Space-time Functionalism', proposed by Huggett and Wuethrich, consider the possibility of a common emergence of space-time and matter [16]. The space-time functionalism is of a wide conceptual generality and requires, that 'higher-level entities/properties/states (of space-time) are realized by the lower level ones'. This approach incorporates the idea of different levels of reality, also worked out by the author of this paper [12].

In this paper, a dual space-concept and the compositeness of particles in its eigenspace represent the main ideas.

The structural analogies between the levels of reality such as the molecular and the atomic level are considered. It is intended, to find a description of 'intrinsic' properties of particles such as spin, mass and magnetic moment at the next deeper, the subatomic level. For this purpose, the composition of particles from subcomponents, circulating masses and charges, is assumed.

This article is organized as follows:

In section two the role of space-time at different levels of reality will be discussed, down to the level where linear space seems to disappear and only circular spaces remain.

Section three contains the introduction of a particle model without translation, having only 'static' properties in space-time.

In section four the translation of a single particle model is investigated, which is able to generate 'quanta' of space and time.

The object of section five is the emergence of linear distances between several particles in relative translation.

The definition of the self-energy of circulating charges is given in section six. Section seven contains the properties of birotons as models of leptons, in particular the derivation of a mass quantum and of gyromagnetic properties.

In section seven it will be discussed, whether isolated subcomponents of particles could exist and possibly play a role in cosmology.

## 2 The different appearances of space-time

The term 'space' is not restricted to geometric constructs with coordinates measurable in length units. Various branches of science use phase spaces, function spaces and other non-geometric types of spaces. In this article, state spaces will be studied at the molecular, the atomic, and the subatomic level.

We remark, that state spaces can also be defined at higher levels, including the level of living systems and of social communities, see the 'dual space concept' in [12] and [13].

At the *molecular level*, a molecule is able to perform different vibrations and rotations. This rotational-vibrational state space is not accessible to single atoms. Only *several* atoms in stable chemical bounds can create these states. The entirety of rotational - vibrational states span a 'common state space' of molecules, not populated by single atoms. Atoms 'do not exist' in the common state space of molecules.

At the *atomic level*, an analogous relationship can be observed. Atoms have states of electronic excitation of their shells, these states span a characteristic common state space. This state space is populated by single atoms or ions, but inaccessible to free electrons or other isolated particles. Particles 'do not exist' in the common state space of atoms. The reason is, that single particles cannot realize this kind of electronic excitation. Only the continual interaction of *several* particles can create these states.

At the *subatomic level*, we suppose an analogous picture. Particles are assumed to consist of subcomponents, called 'rotons'. Several rotons have to interact in order to generate a particle and to appear and to translate in its common state space, the space-time. Single, separated mono-rotons are unable to perform linear translational motions, they 'do not exist' in space-time. Mono-rotons are restricted to a circular space, called basic space, which is invisible to observers and inaccessible to direct experiments.

The molecular, atomic and subatomic level are shown in Fig. 1. A level 0 is added for logical reasons, where only single rotons, the subcomponents of particles, are existing. A transition from this subparticle level 0 to the particle level 1 is not only connected with the emergence od particles, but also the common state space of particles, the space-time, would energe. Space-time emerges together with the development of particles.

From this point of view, space-time appears as the state space of translational states of particles, caused by the coupling of their subcomponents, the rotons.

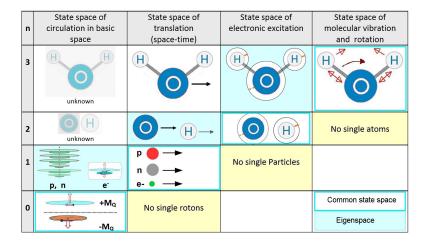


Figure 1: State spaces at the molecular, atomic and subatomic level. The states of complex systems require the interaction of several components and cannot be realized by single, isolated components. Matter at the subatomic level behaves analogously. Subcomponents of particles are not existent in space-time, the translational state space of particles. The yellow colored fields indicate the non-existence of components in the corresponding state space of the systems, consisting of these components.

Rotons bound to a particle structure generate in basic space the intrinsic properties of particles such as mass, spin and magnetic dipole moment. These properties have a circular origin, invariant mass in space time for instance comes from rotational energy and from the self-energy of circulating charges in basic space. These circular properties of an individual particle have the origin in the 'eigenspace' of that particle, where eigenspace means a special case of basic space.

The second column of Fig. 1 shows the different roles of space-time at different levels of reality.

In the first row with level number n=3 space-time represents the domain of translation of molecules. Space-time serves at this level merely as a 'container' for the motion of molecules, the same as for aerosols, birds or any macroscopic bodies.

In the second row (level n=2), space time is the field of translation of atoms, similar as in the first row to molecules. However, electrons show in space time a special motion which has to be described by quantum mechanics, which was first demonstrated by Schrödinger [24]. Simple models resembling a planetary system with motions of atomic electrons on a circle (Sommerfeld [26]) or an ellipse (Bohr [4]) turned out to be unsufficient.

In the third row (level n=1) the particles live in space time as the domain of translational motion, successfully described by Quantum Mechanics. The space time dynamics of particles includes corpuscle-like and wave-like modes of

motion and a multitude of different interactions. Space time at this level plays a complex role, it doesn't represent a 'container', it realizes the 'common state space' of particles.

In the last row (n=0) subcomponents of particles, called rotons, 'do not exist' in space-time. We assume, that at this level reality is represented outside space time by single, unbound rotons living in circular spaces. This unusual extension of reality into inobservable regions corresponds - at least in part - to the usual assumption of virtual particles populating the physical 'vacuum'. Single Monorotons could also play a role in the explanation of dark matter and dark energy, this seems to be a justifiable conjecture.

Fig. 1 suggests the question, why translation represents the most important characteristics of particles in space time and which mechanism generates the capability of linear translation by the coupling of rotons with a purely circular structure. These questions will be discussed in the next section.

## 3 Particle models without translation

The 'biroton' is intended to represent the model of a Dirac-particle in its rest frame. The biroton consists of two rotons with opposite spin direction. The two partial spins  $\hbar$  and  $-\hbar/2$  of the rotons add up to the total spin  $\hbar/2$  of the biroton. The two partial masses of the rotons are nearly equal, see Fig. 2.

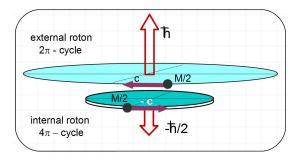


Figure 2: The structure of a biroton, the model of a Dirac - particle (left chiral, spin up). The total spin  $\hbar/2$  corresponds to the spin of a fermion, in Quantum Mechanics to the spin component  $s_z$  in the direction z of the spin axis.

The circular velocities +c and -c of the roton masses correspond to the eigenvalues in the Dirac – theory belonging to the 'Zitterbewegung'. The symmetry of partial masses and the asymmetry of spin components of the rotons reveal some kind of internal supersymmetry between them. The magnitude of the total spin of the biroton  $\hbar - \frac{\hbar}{2} = \frac{\hbar}{2}$  corresponds to the quantum number  $s = \frac{1}{2}$  of fermions. The quantum number  $s + 1 = \frac{3}{2}$  would require parallel spins for both rotons, this is not considered to represent a particle model.

The circulation planes of the rotons represent two separate anticommutative two-dimensional vector spaces. Gamma-matrices serve as unit vectors. The

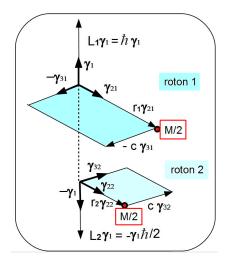


Figure 3: The coordinate schema of a biroton. A two-dimensional circulation plane per roton (colored areas), the common spin axis and the time coordinate result in six dimensions for the biroton. The radius and the tangential momentum of each roton span a noncommutative vector space. The two circulation planes do not belong to the same cylindrical space, because between vectors in  $(\gamma_{21}, \gamma_{31})$  and vectors in  $(\gamma_{22}, \gamma_{32})$  doesn't exist any mathematical operation.

two two-dimensional circulation planes, the common spin axis and the time coordinate result in a total of six dimensions per biroton, see Fig. 3.

The angular momenta of the rotons have a semiclassical definition (all quantities are vectors, we omitted the unit vectors):

$$L_1 = p_{T1}r_1 = \hbar \; ; \quad L_2 = p_{T2}r_2 = -\hbar/2$$
 (1)

$$p_{T1} = M_1 c;$$
  $p_{T2} = -M_2 c$  (2)  
 $r_1 = \hbar/M_1 c;$   $r_2 = \hbar/2M_2 c \approx r_1/2$  (3)

$$r_1 = \hbar/M_1c;$$
  $r_2 = \hbar/2M_2c \approx r_1/2$  (3)

The rotational energy  $\frac{1}{2}Mc^2$  of the circulating mass M is preserved and appears as invariant energy  $E_0 = m_0 c^2$  in space-time.

The cycle length  $l_c$ , the circumference of the external roton, represents the minimum length of the particle model in basic space. The cycle time  $t_c$  is the corresponding minimum time interval.  $A_c$  represents the cycle action of the particle model, equal to Plancks constant h.

$$E_{rot} = \frac{1}{2}Mc^2 = m_0c^2 = E_0 \tag{4}$$

$$l_c = 2\pi r_1 = \frac{h}{M_1 c} \approx \frac{hc}{m_0 c^2} = \frac{hc}{E_0}$$
 (5)

$$t_c = \frac{l_c}{c} \approx \frac{h}{E_0}; \qquad A_c = E_0 t_c = h \tag{6}$$

This conservation law for the energy, see eqn. 4, implies a halving of the circulating mass M:

$$\frac{1}{2}M \approx M_1 \approx M_2 \approx m_0 \tag{7}$$

Slight differences between the roton masses  $M_1$  and  $M_2$  due to differences of the self-energies of circulating charges are neglected in the equations (5) and (7).

## 4 Linear translation of a single biroton

#### 4.1 Rotational and translational events

The circulation of masses and charges of a biroton in basic space represents a geometric quantization in cycles, that means  $2\pi$  radians for the external and  $4\pi$  radians for the internal roton. In terms of the probability theory, one cycle quantum equals one 'rotational event'. The circular motion generates the constant spin contributions of the rotons, this is a deterministic process. The angular momenta  $L_1$  and  $L_2$  doesn't show any fluctuations.

Besides deterministic rotational events generating the spin we assume the occurence of an additional type of stochastic events called translational events. Such additional events occur without disturbing the continual spin rotation. The combination of rotational and translational events represents the simultaneous propagation of the biroton in the circular basic space and in the linear spacetime by the same distance, the cycle length  $l_c$ . At models of photons, all events are combinational events. The cycle length appears as circumference of the external roton as well as wave length of the photon. The velocity of light c represents a circular velocity in basic space and linear velocity in space-time. Combinational events occur in models of massive particle only with a certain probability.

We define

 $\beta_i^2$  as the probability, that the next rotational event of the roton i is accompanied by a translational event and

 $R_i^2 = 1 - \beta_i^2$  as the probability, that the next rotational event of the roton i is without translation. Thus one obtains

$$R_i^2 + \beta_i^2 = 1; \quad i = 1, 2$$
 (8)

The two rotons of a biroton are completely undisturbed by translational events, if simultaneously in both rotons no translational events occur. The combined probability  $\mathbb{R}^2$  for this event of pure rotation is equal to the product

$$R^{2} = R_{1}^{2} * R_{2}^{2} = (1 - \beta_{1}^{2})(1 - \beta_{2}^{2})$$
(9)

The corresponding translational probability  $\beta^2 = 1 - R^2$  depends on the translational probabilities  $\beta_i^2$  of both rotons:

$$\beta^2 = 1 - R_1^2 * R_2^2 = \beta_1^2 + \beta_2^2 - \beta_1^2 \beta_2^2 = \beta_1^2 + R_1^2 \beta_2^2$$
 (10)

The translational probability  $\beta^2$  of the biroton cannot increase to values above unity. This is true, even if both rotons have probabilities  $\beta_i^2 \approx 1$  and the sum of their probabilities exceeds 1. This behavior is well known from the Special Relativity Theory (SRT) and describes the relativistic addition of two orthogonal velocities. We interpret the components  $\overrightarrow{\beta_i}$  in Minkowski spacetime as dimensionless vector-like measures of orthogonal components  $\overrightarrow{p_i}$  of the particle's momentum  $\overrightarrow{p}$ :

$$\overrightarrow{\beta_i} = \frac{c\overrightarrow{p_i}}{|E|}; \ i = 1, \ 2; \ \overrightarrow{p_1} \perp \overrightarrow{p_2}$$
 (11)

$$\overrightarrow{\beta} = \frac{c \, \overrightarrow{p}}{|E|} \tag{12}$$

E is the total energy of the biroton in space-time,  $E_0$  the invariant energy (rest energy) in space-time, originated by the rotational energy of circulating masses and charges.

We name the quantity  $\overrightarrow{\beta_i}$  'transil' of the roton *i*. It represents a dimensionless vector while the transil square  $\beta_i^2$  is a probability.

The relation (10) gives  $\overrightarrow{\beta_1} \perp \overrightarrow{\beta_2}$  and therefore an orthogonal relationship exists between  $\overrightarrow{p_1}$  and  $\overrightarrow{p_2}$ . We choose  $\overrightarrow{p_1}$  as the component in spin direction (or opposite to the spin, depending on the sign) and  $\overrightarrow{p_2}$  as the component orthogonal to the spin axis. The two momentum components are sufficient to describe translation in three spatial dimensions, because the direction 'orthogonal to the spin axis' defines an area, not a one-dimensional direction.

The scalar quantity R is called 'rotil' of the particle and can be interpreted as the relation between invariant energy and total energy:

$$R = \frac{E_0}{|E|}; \qquad E = \frac{1}{R}E_0 \tag{13}$$

The reciprocal of the rotil R is the Lorentz-factor  $\gamma$ .

$$\gamma = \frac{1}{R} = \frac{1}{\sqrt{1 - \beta^2}} \tag{14}$$

In the absence of translational events one obtains  $R^2 = 1$ , this characterizes the static properties of the biroton. The condition  $R^2 = 1$  can be realized only for particle models containing mass quanta such as models of massive leptons and hadrons.

Photons travel always with the velocity of light parallel or anti-parallel to the spin axis, i.e. one has for the photon model

$$R^{2}(ph) = R_{1}^{2}(ph) \equiv 0 \; ; \quad R_{2}^{2}(ph) \equiv 1$$
 (15)

$$\beta^2(ph) = \beta_1^2(ph) \equiv 1; \quad \beta_2^2(ph) \equiv 0$$
 (16)

The invariant energy of photons is zero, because they do not contain circulating mass quanta.

Using the definitions (12) and (13), one obtains a correspondence between the probabilistic equation derived in basic space and the relativistic energymomentum relation valid in Minkowski' space-time:

$$R^2 + \beta^2 = 1 (17)$$

$$\left(\frac{E_0}{E}\right)^2 + \left(\frac{cp}{E}\right)^2 = 1; \quad E_0^2 + (cp)^2 = E^2$$
 (18)

The double role of equation (17) qualifies it as a bridge between the particle model in basic space - the biroton - and the observed particle in space-time.

### 4.2 Linear and circular distances

A biroton without translation, 'at rest' in space-time, travels by circular motion one cycle length  $l_c$  per cycle time  $t_c$  (see the definitions (5) and (6).

During translation with the uniform average velocity  $v = \beta c$  the motion of the biroton is characterized by circular as well as linear quantities of the dimension 'length'. The cycle length  $l_c$  gets in the average two orthogonal components, the circular 'Lorentz interval'

$$c\tau_c = Rct_c = Rl_c \tag{19}$$

and the linear spatial interval

$$x_{\beta} = \beta c t_{c} = \beta l_{c} \tag{20}$$

The quantity  $\tau_c = Rt_c$  represents the proper time of the biroton. The two orthogonal components of  $l_c$  add quadratically:

$$l_c^2 = (c\tau_c)^2 + x_\beta^2 = (R^2 + \beta^2)l_c^2$$

The 'Lorentz interval'  $c\tau_c$  corresponds to the space-time interval ds in the SRT. One obtains

$$(c\tau_c)^2 = l_c^2 - x_\beta^2 \tag{21}$$

$$ds^2 = c^2 dt^2 - dx^2 \tag{22}$$

The very small but finite intervals related to basic space models correspond to infinitesimal quantities in SRT.

It is suitable to differentiate between three cases of the spatial interval  $x_{\beta}$ :

- $x_{\beta} = l_c$  the 'lightlike' case: The linear interval per cycle equals the full circular cycle length, proper time is zero,  $\tau_c \equiv 0$  and  $\beta = 1$ , identically with the characteristics of a photon.
- $x_{\beta} < l_c$  the 'timelike' case: The linear interval per cycle is smaller than the full circular cycle length, proper time is positive nonzero,  $\tau_c > 0$  and  $\beta < 1$ .
- $x_{\beta} > l_c$  the 'spacelike' case is excluded if only a single particle is under consideration. This would result in a negative proper time  $\tau_c < 0$  which would be unphysical.

# 4.3 Combinational events, the local and nonlocal mode of translation

The 'transil' quantities  $\overrightarrow{\beta_1}$  and  $\overrightarrow{\beta_2}$  defined in (12) determine the orthogonal directions of the two linear components of the velocity  $\overrightarrow{\beta c}$  and the average of the magnitude.

The translation of the biroton appears as an intermittent process. During combinational events, jumps with the linear velocity of light  $\overrightarrow{c}$  are performed in addition to the rotational motion with the circular velocity c. Combinational events alternate with purely rotational events, where the particle is 'at rest' in space-time. The linear velocity during purely circular cycles is zero. The average linear velocity of this intermittent process amounts to  $\overrightarrow{\beta c}$ .

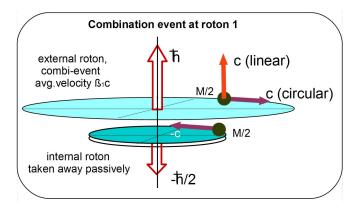


Figure 4: Combinational event with a linear step parallel to the spin axis with the velocity c. The external roton initiates the linear motion, the internal roton is passively taken away connected to the external roton. The 'rotil connection' is symbolized by a strong common spin axis. The linear step contributs an average velocity component  $\beta_1 c$ .

The Figures (4) and (5) show a geometric schema of combinational events with linear steps parallel and perpendicular to the spin axis.

The two types of combinational events represent the 'local mode' of translation, because the two rotons travel together and the biroton has always a well defined position in space-time. The active roton takes away the passive one due to a strong connection by a common spin axis. The probabilistic equation  $R^2 + \beta_1^2 + \beta_2^2 R_1^2 = 1$  represents a variant of equation (8) and can be linearized in order to use the split of the transil  $\overrightarrow{\beta}$  into two components. The translation represents in the average a uniform motion with the constant velocity  $\beta c$ , if external influences (forces) are absent. If  $\beta$  changes from step to step, acceleration occurs. If the time dependence is different for  $\beta_1$  and  $\beta_2$ , the trajectory of the biroton becomes curved.

A second 'nonlocal mode' of translation has no common spin axis, whereas the directions of the roton axes remain parallel. The connection between the

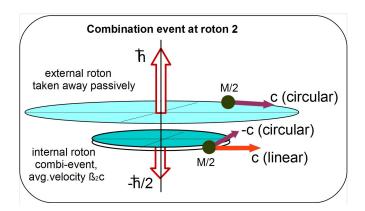


Figure 5: Combinational event with a translational step perpendicular to the spin axis with the velocity c. The internal roton initiates the linear motion, the external roton is passively taken away connected to the internal roton. The 'rotil connection' is symbolized by a strong spin axis. The linear step contributes an average velocity component  $\beta_2 c$ .

rotons is given by a common transil  $\overrightarrow{\beta}$  realized by synchronous translational events. The existence of such a nonlocal mode can be described mathematically by a different linearization of the probabilistic equation

$$R^2 + \beta^2 = R_1^2 R_2^2 + \beta^2 = 1.$$

The two possible linearizations are compared in the following.

In the *local mode* the rotons are connected by the common rotil R, and the transil  $\beta$  is splitted into the spatial components  $\beta_1$  and  $\beta_2$ :

$$R^2 + \beta_1^2 + \beta_2^2 R_1^2 - 1 = 0 (23)$$

$$(R\gamma_0 + \beta_1\gamma_1 + \beta_2R_1\gamma_2 - 1)(R\gamma_0 + \beta_1\gamma_1 + \beta_2R_1\gamma_2 + 1) = 0$$
 (24)

In the nonlocal mode, the rotons are connected by the common transil  $\beta$ . The rotil R is splitted into  $R_1 = E_0/E_1$  and  $R_2 = E_0/E_2$ , that results in different roton energies  $E_1$  and  $E_2$  as well as different proper times  $\tau_i = R_i t$ . The linearization reads

$$R_1^2 R_2^2 + \beta^2 - 1 = 0 (25)$$

$$(R_1^2 \gamma_0 + \beta \gamma_{\parallel} - 1)(R_2^2 \gamma_0 + \beta \gamma_{\parallel} + 1) = 0$$
 (26)

The two linear expressions in equations (24) and (26) can separately set to zero. Please note, that the linear expressions in the local mode are symmetrically for both rotons. That means, the rotons are fixed to identical coordinates in space-time.

In the nonlocal mode, however, one obtains two different equations, one for each roton. This allows different positions and paths for the rotons in space-time. The average direction is identical for both rotons and determined by the symbolic vector  $\gamma_{\parallel}$ , anticommuting with  $\gamma_0$ , the unit vector of the time coordinate. A more detailed theory could  $\gamma_{\parallel}$  describe as result of a linear combination of the unit vectors  $\gamma_1$  and  $\gamma_2$ .

We remark, that only the compositeness of the particle model allows this type of linearization like in equation (26). A non-composed, structure-less particle model enables only the linearization according to equation (24).

The biroton in local mode is observable in space-time as a corpuscle, it has 4 dimensions in this mode. The biroton in nonlocal mode is not directly observable, it appears as probability wave in space-time. Two rotons take part in this wave, presumably the biroton may appear as a couple of two parallel waves. This assumption could help to understand the self-interference of a single particle in a double-slit experiment. The biroton in nonlocal mode has six dimensions in space-time, the same as in basic space. This could be the reason of the inobservability in that mode.

The biroton can change between the two modes in its eigenspace without delay. The change from the nonlocal into the local mode appears for an observer in space-time as an abrupt, mysterious process. Quantum - Mechanics describes the nonlocal mode as a wave packet or a superposition of quantum states. The wave function has to be updated instantly, if the biroton changes its mode of translation. Sometimes this change is interpreted as 'collapse' of the wave function. The wave or wave packet disappears and the particle becomes observable and detectable as a corpuscle on a screen. The biroton as a particle model, changeable between two appearances in space-time having four respective six dimensions, may help to understand the situation.

A symbolic geometric picture of the nonlocal mode gives Fig. 6.

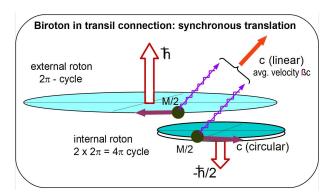


Figure 6: A biroton in nonlocal mode of translation. The spin axes of the rotons are parallel, but not aligned. The connection between the rotons exists in their synchronous translational events with the velocity c, which appear in space-time as two parallel probability waves. Their average velocity is  $\beta c$ .

The number of dimensions of the biroton in basic space and in space-time is shown in Fig. 7.

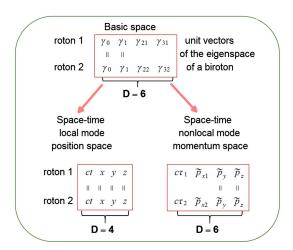


Figure 7: Dimensionality of the biroton in basic space and in space-time. The biroton has in basic space six dimensions,  $2 \times 2$  from the two circulation planes, one from the direction of the spin axis and a time coordinate. In local mode, a reduction to 4 dimensions takes place due to the rotil coupling between the rotons. The nonlocal mode preserves the original six dimensions. Two waves have four dimensions each in momentum space, two of them are identically.

A particle model performs translation in its 'common state space', see Fig.1. This turns out to be a stochastic process, an overlay onto the deterministic continual spin rotation. An overview over the four states, which constitute the intermittent process of translation, is given in Fig. 8.

### 4.4 The probability wave

A few remarks should be made on the question, whether the particle model defined in basic space can contribute to the nature of 'probability waves', used by Quantum Mechanics for the definition of quantum states.

Because we introduced the probabilities  $R^2$  and  $\beta^2$  at the very beginning of this article (see equation (8), one can derive some properties of the probability wave.

The entirety of translational events is of a stochastic nature and occurs with the probability  $\beta^2$ . During a combinational cycle, a translational step with the linear velocity of light c and the cycle distance  $x_c$  is passed by the biroton. During subsequent pure rotational cycles the linear velocity is zero. The next combinational cycle follows after  $n_a = 1/\beta$  cycles in the average, such that the average velocity becomes  $v = \beta c$ , averaging the one-cycle velocity c over  $n_a$  cycles. The translation time during  $n_a$  cycles is  $t = n_a t_c$ , where  $t_c$  means the cycle time, defined in eqn. (6).

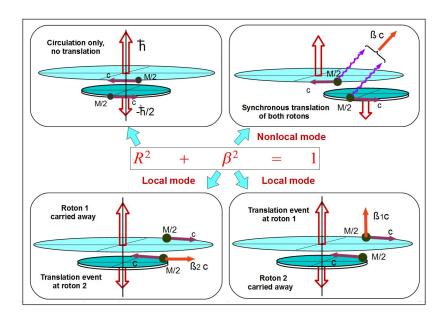


Figure 8: States of the biroton during the process of translation. The indicated linear velocities such as  $\beta c$  and  $\beta_i c$  are averages. The linear length interval generated in combinational cycles is always constant and equals the cycle length of the circular motion.

Quantity	(	One cycle	$n_a = 1/\beta$ cycles
	rotat.c.	combinat. cycle	sum or average
time interval	$t_c$	$t_c$	$t = n_a t_c = t_c / \beta$
linear distance	0	$x_c = ct_c$	$x = x_c = vt$
linear velocity	0	$c = \frac{x_c}{t_c}$	$v = \frac{x}{t} = \beta c$
linear momentum	0	$\frac{ \vec{p} }{\beta} = \frac{ E }{c} = \frac{m_0 c}{R}$	$\left  \overrightarrow{p} \right  = \frac{ E }{c} \left  \overrightarrow{\beta} \right $
energy, classic	$E_0$	$E_0 + \frac{1}{2}m_0c^2$	$E_0 + \frac{1}{2}m_0v^2$
energy, relativist.	$E_0$	$E_0 + \frac{1}{R+R^2} m_0 c^2$	$E_0 + \frac{1}{R + R^2} m_0 v^2 = \frac{E_0}{R}$

Figure 9: Quantities of intermittent translation. The expressions for quantities related to one cycle in combinational events contain the velocity of light c, while the quantities representing a sum or the average over a number of cycles contain the average velocity  $v = \beta c$ .

The linear cycle distance

$$x_c = ct_c = \frac{hc}{E_0} = \frac{h}{m_0 c} = \lambda_{Compton}$$
 (27)

remains constant over  $n_a$  cycles, because no further increase occurs.  $x_c$  equals the Compton wave length  $\lambda_{Compton}$  of the particle.

During rotational cycles, the linear distance  $x_c$  does not change. As a function of the average velocity v one obtains after  $n_a$  cycles  $x = x_c = vt$  (see Fig. 9).

The identical magnitudes of the circular cycle length  $l_c = hc/E_0$  defined in eqn. (5) and the linear cycle distance  $x_c = hc/E_0$  illustrate the connection of basic space and space-time. The invariance of the energy  $E_0$  and of the time interval  $t_c$  represent a typical example of the dual space concept.

The invariant energy  $E_0$  is generated in form of rotational energy, see eqn. (4). In a combinational cycle, a supplement to  $E_0$  develops, known as kinetic energy. A first intuitive assumption leads to an additional energy of  $\frac{1}{2}m_0c^2$ , corresponding to the invariant energy of one roton, active in the combinational cycle. By averaging over  $1/\beta$  cycles, the average additional energy would become  $\frac{1}{2}m_0v^2$ , the kinetic energy in classical, non-relativistic physics. Thus one has to look for the correct relativistic additional energy. One gets  $m_0c^2/(R+R^2)$  instead of  $m_0c^2/2$ . Thus the kinetic energy is not an invariant quantity. Replacing c by  $v = \beta c$  one obtains a relativistic formula for the kinetic energy

 $E_{kin} = \frac{m_0 v^2}{R+R^2}$ . The sum of invariant and kinetic energy gives the correct expression of the total energy  $E = E_0/R$ :

$$E_0 + E_{kin} = m_0 c^2 + \frac{m_0 v^2}{R + R^2} = \frac{E_0}{R}$$
 (28)

A proof of equation (28) uses division by  $m_0c^2$ :

$$1 + \frac{\beta^2}{R + R^2} = 1 + \frac{1 - R^2}{R(1 + R)} = 1 + \frac{(1 - R)(1 + R)}{R(1 + R)} = 1 + \frac{1 - R}{R} = \frac{1}{R}$$

A graphical visualization of length and time intervals during the intermittent process of translation is given in Fig.10. As an example, we set  $\beta = 1/3$  and  $n_a = 1/\beta = 3$ .

The average time between two translational events is  $t_c/\beta=3t_c$ , during this time the biroton travels one cycle length  $x_c=ct_c$ . The red zig-zag line in Fig. 10 does *not* represent the probability wave. Instead, the wave could be generated as an envelope of several translational events. The de Broglie wavelength  $\lambda$  and the wave period  $t_{\lambda}$  amount to

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{|E|\beta} = \frac{R}{\beta}x_c \tag{29}$$

$$t_{\lambda} = \frac{\lambda}{v} = \frac{R}{\beta^2} \frac{x_c}{c} = \frac{R}{\beta^2} t_c \tag{30}$$

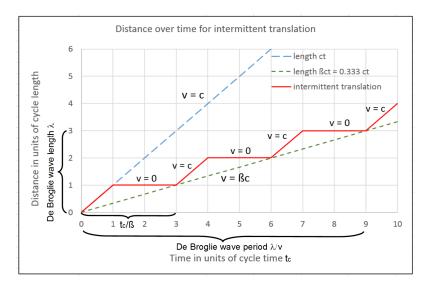


Figure 10: Distances over time for intermittent translation. A schematic view is given for  $\beta = 1/3$  as an example. The equidistant translational events represent a simplification. It shows only the average of a stochastic process, The quantization of the translational motion leads to a quantized generation of linear space.

where  $x_c = hc/E_0$  represents the linear cycle length, see eqn (27). The example drawn in Fig. 10 shows the de Broglie wavelength at

 $\lambda = 3x_c = x_c/\beta$  and the wave period at  $\lambda/v \approx 9t_c = t_c/\beta^2$ , we approximated  $R = 0.9428 \approx 1$  for graphic convenience. The geometric interpretation of the de Broglie wavelength  $\lambda$  depends on the parameter  $\beta$ , the average relative velocity of translation.

In Fig. 12 a Gaussian standard distribution (normal distribution, bell curve) is compared with a squared cosine and sine function as a simple example of a probability density. We suppose, that stochastic translational events are normal distributed over time and such standard distributions are repeated periodically, triggered by the deterministic rotational events.

It seems to be imaginable, that the probability density of Quantum Mechanics represents a convenient approximation of a periodic Gaussian distribution. Two periodic Gaussian distributions, originated from two rotons of the particle model, could result in the same total probability density as the sum of sin<sup>2</sup> and cos<sup>2</sup> functions. It remains unclear, however, why the square root of the probability densities, the probability amplitudes, achieve a special importance in Quantum Mechanics, while the square root of a Gaussian distribution has no special meaning in the probability theory. The two rotons in the structure of a biroton would suggest to assign one probability amplitude to one of the rotons.

The form of the standard distribution cannot be derived currently from

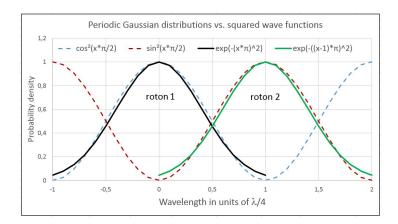


Figure 11: A periodic Gaussian distribution compared with a  $\cos^2$  wave function. The sum of squared sin and  $\cos$  wave functions represents an example for a probability density in Quantum Mechanics.

the particle model. However, the central limit theorem could explain, that the distribution of any random variable converges to a normal distribution as the number of samples increases. The main issue of the proposed intermittent translation model is therefore the fact, that combinational cycles originating a linear jump with the velocity of light are randomly distributed around an average distance of  $1/\beta$  cycles.

The essential result of this section is the emergence of quanta of space and time, the linear cycle distance  $x_c$  and the cycle time  $t_c$ , generated by a single particle during translational events.

An extension to several particles will be discussed in the next section.

## 5 The translation of several particles

## 5.1 The relativistic addition of velocities

We consider two independent particles (1) and (2), and the equation (17) holds for each of them:

$$R^2(i) + \beta^2(i) = 1; \quad i = 1, 2.$$

Two separated particles generate by translation two insulated pieces of spacetime, characterized by the transils  $\overrightarrow{\beta(1)}$  and  $\overrightarrow{\beta(2)}$  as well as the rotils R(1) and R(2). A space-time distance between two particles can be defined by uniting their probability spaces (sample spaces). We consider the case of parallel translation of two particles,  $\overrightarrow{\beta(1)} \parallel \overrightarrow{\beta(2)}$ .

The intuitive connection leads to R(1) \* R(2) for the rotil and  $\overline{\beta(1)} + \overline{\beta(2)}$  for the transil of the connected system. Unfortunately, the squared quantities

 $R^2(1) * R^2(2) + (\beta(1) + \beta(2))^2$  do not represent the probability of the united system. In the extreme case of two photons travelling in the same direction, that means

 $\overrightarrow{\beta(1)} \equiv \overrightarrow{\beta(2)} \equiv 1$ , one would get

 $(\beta(1) + \beta(2))^2 = 4$ . Therefore one has to define a normalization constant in order to obtain a complete probability space. We put in

 $R^2(i) = 1 - \, \beta^2(i)$  , the result for the normalization constant of probabilities is

$$R^{2}(1)R^{2}(2) + (\beta(1) + \beta(2))^{2} = (1 + \beta(1)\beta(2))^{2}$$
(31)

Rotil  $R_{\parallel}$  and transil  $\overrightarrow{\beta}_{\parallel}$  of the connected system of two particles have now the form

$$R_{\parallel} = \frac{R(1)R(2)}{1 + \beta(1)\beta(2)} \; ; \; \overrightarrow{\beta_{\parallel}} = \frac{\overrightarrow{\beta(1)} + \overrightarrow{\beta(2)}}{1 + \beta(1)\beta(2)}$$
(32)

$$R_{\parallel}^2 + \beta_{\parallel}^2 = 1 \tag{33}$$

The rotil  $R_{\parallel}$  cannot be used to calculate a proper time, because two independent particles do not have a common proper time.

The expression for the transil  $\overrightarrow{\beta}_{\parallel}$  leads to a formula for the addition of two parallel velocities

$$\overrightarrow{v(1)} = \overrightarrow{\beta(1)c} \parallel \overrightarrow{v(2)} = \overrightarrow{\beta(2)c}$$
. We obtain

$$\overrightarrow{v_{\parallel}} = \overrightarrow{\beta_{\parallel}c} = \frac{\overrightarrow{v(1)} + \overrightarrow{v(2)}}{1 + v(1)v(2)/c^2}$$
(34)

Equation (34) represents the correct formula for the relativistic addition of velocities, known from SRT.

## 5.2 The special Lorentz transformation

A system of two particles provides the opportunity to use one of them as 'reference frame' for the translation of the whole system. We choose the particle with index 1 as reference, the quantities belonging to this reference system are marked with a stroke: R'(1) = 1;  $\beta'(1) = 0$ .

The probability  $R_{\parallel}^2$  that translational steps are completely absent in the connected system has to remain unchanged during the transformation, that means

$$R_{\parallel}^{\prime 2} = R_{\parallel}^2 \ invariant \tag{35}$$

The space-time intervals (Lorentz intervals) of the two particles remain separately invariant.

$$c\tau(1) = cR(1)t = cR'(1)t' = ct' \quad invariant$$
(36)

$$c\tau(2) = cR(2)t = cR'(2)t'$$
 invariant (37)

Rotil and transil of particle 2, which performs now the translational motion completely, get after the transformation the form

$$R'(2) = R_{\parallel} = \frac{R(1)R(2)}{1 + \beta(1)\beta(2)} \tag{38}$$

$$\overrightarrow{\beta'(2)} = \overrightarrow{\beta_{\parallel}} = \frac{\overrightarrow{\beta(1)} + \overrightarrow{\beta(2)}}{1 + \beta(1)\beta(2)}; \quad \overrightarrow{\beta(1)} \parallel \overrightarrow{\beta(2)}$$
(39)

The time coordinate after the transformation is given by

$$t' = \frac{R(2)}{R'(2)}t = \frac{(1+\beta(1)\beta(2))t}{R(1)} = \frac{t+v(1)x(1)/c^2}{\sqrt{1-\beta^2(1)}}$$
(40)

The particle 1, now the 'reference frame', is at rest in space-time and therefore its spatial interval x is zero:  $x'(1) = \beta'(1) c t' = 0$ 

The x - coordinate of particle 2 after transformation, representing the spatial distance between particle 1 and 2, is given by

$$\overrightarrow{x'(2)} = \overrightarrow{\beta'(2)ct'} = \frac{\overrightarrow{x(1)} + \overrightarrow{x(2)}}{\sqrt{1 - \beta^2(1)}} = \frac{\overrightarrow{x(2)} + \overrightarrow{v(1)}t}{\sqrt{1 - \beta^2(1)}}$$
(41)

The formulas (40) and (41) represent special Lorentz transformations in 1 + 1 dimensions.

For the reverse transformation from the frame with stroke back to the original frame one gets nearly identical formulas. Any of the two particles can serve as 'reference frame', the other particle can perform all the translational steps. The development of linear pieces of space-time is not fixed to the structure of a distinguished particle.

The essential result of this section is the emergence of intervals of space and time between different particles, generated by their relative translation. Presumably the entirety of such intervals gives rise to the development of a continuum of general space-time.

## 6 The self-energy of circulating charges

#### 6.1 The Coulomb law in a circular space

In classical electrostatics, the Coulomb energy  $E_C$  in a linear distance d from a point charge amounts to

$$E_C^{SI} = k_e \frac{\alpha \hbar c}{d} = k_e \frac{e^2}{d}; \quad E_C^G = \frac{\alpha \hbar c}{d} = \frac{e^2}{d}$$
 (42)

where  $\alpha$  means the fine structure constant. The Coulomb energy can be expressed in SI or in Gaussian (G) units. We use the Gaussian system of quantities

and units, thus the Coulomb constant becomes  $k_e = 1$ . Linear distances d can be defined only in space-time, where two or more particles are present. In the eigenspace of a single particle, d represents a circular distance such as the cycle length  $l_c$  or a multiple of  $l_c$ . Consequently, we consider  $d = 2\pi r_1 = 4\pi r_2$  as the self-distance of a charge circulating with the radius  $r_1$  or  $r_2$ , see the definitions in eqn. (2) and Fig. 3. The Coulomb energy

$$E_{C1} = \frac{\alpha \hbar c}{2\pi r_1} = \frac{\alpha}{2\pi} M_1 c^2 \tag{43}$$

becomes the 'self-energy' of a charge circulating together with the mass  $M_1$  of the first roton. The quantity 'self-energy' does not exist in classical electrostatics.

The roton is divided into 2k parts, called 'rings', and the charge may be attached to the ring mass  $M_r = M_1/2k$  of one ring only, where k is the ring parameter. One gets the self-energy  $E_{Cr}$  of the charged ring

$$E_{Cr} = \frac{k\alpha}{\pi} * \frac{1}{2k} M_1 c^2 = kz * M_r c^2$$
 (44)

$$z = \frac{\alpha}{\pi} = 2.322819465 \times 10^{-3} \tag{45}$$

The total energy  $E_r$  of the ring in basic space comprises now the rotational energy  $E_{rot,r} = \frac{1}{2} M_r c^2$  and the self-energy  $E_{Cr}$  of the charge.

The energy conservation leads to corresponding expressions in space-time, however, there are two possibilities of energy conservation:

1. The charge is *fixed* to the ring mass (index f) such that its self-energy gets halved together with the ring mass:

$$E_{r,f} = E_{rot,r} + \frac{1}{2}E_{Cr} = \frac{1}{2}M_rc^2(1+kz) = m_rc^2(1+kz)$$
 (46)

2. The charge circulates *independent* of the ring mass (index s), and the self-energy  $E_{Cr}$  is conserved separately without halving:

$$E_{r,s} = E_{rot,r} + E_{Cr} = \frac{1}{2}M_rc^2 + kz * M_rc^2 = m_rc^2(1 + 2kz)$$
 (47)

$$E_{r,s} = m_r c^2 (1 + kz) + kz * m_r c^2 = E_{r,f} + \Delta E_r$$
(48)

The total energy in the case of "independent" charges equals the total energy for fixed charges plus a small quantity  $\Delta E_r$ . This surplus-amount of energy plays an important role at the calculation of the anomalous dipole moment of leptons, see section 7.2.

We assume, that single electromagnetic (em) charges are always *independent* charges, while fluctuating charge pairs and the group of weak charges are *fixed* charges, see the next sections and Fig.13.

## 6.2 Energy factors of circulating charges

The factor (1 + kz) in equation (46) increases the energy of a charged ring and is named 'energy factor'. The precise value of an energy factor depends on the

number of cycles, during which the self-interaction of the charge continues. For two cycles, one obtains the factor

$$d_k = 1 + kz(1+kz) = 1 + kz + (kz)^2$$
(49)

Different types of charged rings differ by the number of cycles, after that the charge jumps to another ring, where the self-interaction starts again. Four types and its energy factors are shown in Fig. 12

Self-energy factors of a circulating charge, k ring parameter 
$$w_k = \underbrace{1 + kz + (kz)^2 + (kz)^3 + (kz)^4 + (kz)^5 + \dots}_{b_k} b_k \text{ one cycle}$$

$$\underbrace{d_k + (kz)^2}_{d_k} b_k \text{ one cycles}$$

$$z = \frac{\alpha}{\pi}$$

$$a_k \text{ three cycles}$$

$$w_k = \sum_{i=0}^{\infty} (kz)^i = \frac{1}{1-kz} \text{ ; } w_k \text{ max. self-energy}$$

Figure 12: Energy factors of circulating charges in dependence on the number of cycles of continuous self-interaction.

Rings with one-cycle charges and energy factors  $b_k = 1 + kz$  appear always pairwise as electrically neutral entity and are called 'fluctuating charge pairs'. They are responsible for the coupling between rings and their self-energy is conserved together with the rotational energy of the ring mass according to eqn. (46). FLuctuating charge pairs compensate each other within the particle structure, they are inobservable in space-time.

Infinitly circulating charges represent single electromagnetic (em) charges, they have the energy factor  $w_k$  and their self-energy is separately conserved according to equations (47) and (48). Single em charges can be observed in space-time as a property of a 'charged' particle.

Double rings carrying two charges of the same sign represent weak charges, a total of four weak charges is electrically neutral. Four weak charges give the correct weak coupling strength relative to that of an em charge, see Fig.13.

The energy factor of the fourfold ring with alternating charges is  $u_k < 1$ , it reduces the rotational energy of the circulating masses:

$$u_k = 1 - kz(1 + kz(1 - kz(1 + kz(1 - kz(1 + kz(1 - kz)))))) = (50)$$

$$u_k = 1 - kz - (kz)^2 + (kz)^3 + (kz)^4 - (kz)^5 - (kz)^6 + (kz)^7 + \dots$$
 (51)

The ring with four charges with alternating signs complements the group of electroweak charges, it compensates the big self-energy of the four weak charges.

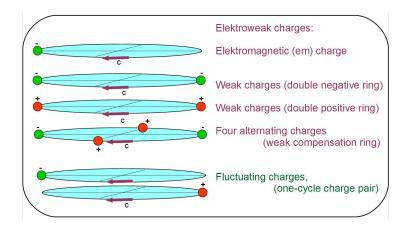


Figure 13: Ring types carrying integer charges which realize different interaction capabilities. The electromagnetic (em) charge at the top of the figure determines the observable charge of a particle. The weak interaction is realized by eight single integer charges located at three rings. Positive and negative weak and fluctuating charges compensate each other internally and do not influence the electromagnetic properties of a particle.

## 6.3 The effective em coupling strength $\alpha_{em,eff}$

The Coulomb energy  $E_C$  of two charged particles according to eqn. (42) represents the electrostatic potential energy between two elementary charges e(1) and e(2) in the linear distance d = x'(2), see eqn. (41). The same amount of Coulomb energy  $E_C$  is generated as self-energy of a single charge in a circular self-distance  $d = 2\pi r_1$ . This energy conservation during the transition from basic space to space-time represents another typical example of the dual space concept. The electromagnetic coupling constant  $\alpha = \alpha_{em}$  is the same in space-time, where two charges e(1); e(2) exist, and in basic space, where a single charge e realizes  $e(1) \equiv e(2) = e$ . One gets in the Gaussian system of units

$$\alpha_{em} = \frac{e(1)e(2)}{\hbar c} = \frac{e^2}{\hbar c} \tag{52}$$

The effective electromagnetic coupling  $\alpha_{em,eff}$  increases with the ring parameter k, it is 'running', related to a roton with 2k rings:

$$\alpha_{em,eff}(k) = \frac{2\pi}{2k}(w_k - 1) = \alpha + \alpha kz + \alpha (kz)^2 + \dots = \alpha w_k$$
 (53)

and the reciprocal becomes, using the sum formula  $w_k = 1/(1-kz)$ 

$$\frac{1}{\alpha_{em,eff}(k)} = \frac{1}{\alpha}(1 - kz): \quad z = \frac{\alpha}{\pi}$$
 (54)

The reciprocal of the effective coupling shows a decrease with increasing ring

parameter k, that means with increasing energy of the particle. This tendency agrees with the behavior of a 'running constant'.

## 7 Birotons as lepton models

# 7.1 Skeleton and dress, the introduction of a mass quantum

The three massive leptons, electron, muon, and tauon (and its antiparticles) can be modelled as composite structures in basic space. However, the order has to be changed into muon, electron and tauon. Exploring the basic space models, the ring parameter k becomes 8, 16, and 32 for muon, electron and tauon, as will be demonstrated within this section. Moreover, the ring mass of the muon  $M_r(\mu)$  appears also as a building stone in the models of electron and tauon, and therefore we define this ring mass as "mass quantum"  $M_Q$  in basic space and  $m_Q = \frac{1}{2}M_Q$  in space-time. The ring parameter k = 8 for the muon implies 2k = 16 rings per roton and 4k = 32 rings per particle, thus we get a preliminary value for the mass quantum from the mass  $m_\mu$  of the muon:

$$m_Q = \frac{1}{2} M_Q \approx \frac{1}{32} m_\mu \approx 3.30 \ MeV/c^2$$
 (55)

The precise value of  $m_Q$  will be derived in section 7.3.

The main contribution to the mass of a particle comes from the circulating mass, they form the 'skeleton' of the particle model. A minor contribution to the mass comes from the self-energy of circulating charges, we call this part the 'dress'. The skeleton masses of a lepton can be expressed by a general formula:

$$m_{lept}^s(k) = (2k \pm 2k) * \frac{k}{8} m_Q ,$$

where 2k represents the number of rings per roton and  $\frac{k}{8}m_Q$  is the ring mass  $m_r$ . The small electron mass forces to assume the existence of negative mass quanta, such that the electron gets the skeleton mass zero due to the internal compensation of two large roton masses.

The ring mass increases from generation to generation by a factor of two, it reaches  $4m_Q$  for the tauon at k=32. The number of rings also increases by a factor of two, that results in a quadratic increase of the number of mass quanta per lepton for muon and tauon:

$$m_{\mu, \tau}^{s} = \frac{1}{2}k^{2}m_{Q} \tag{56}$$

$$m_{\mu}^{s} = \frac{1}{2}8^{2}m_{Q} = 32m_{Q} \tag{57}$$

$$m_{\tau}^s = \frac{1}{2} 32^2 m_Q = 512 m_Q \tag{58}$$

The table in Fig.14 contains numerical values of the skeleton masses. The first column shows the ring parameter k, which also indicates the number n of the particle generation:

$$k = 2^{n+2}$$
; n generation number (59)

	Skeleton masses of massive leptons							
k	$m_{lept}^s(k) = (2k \pm 2k) * \frac{k}{8} m_Q$	MeV/c <sup>2</sup>						
8	$m_{\mu}^{s} = (16 + 16) * m_{Q}$	104. 091						
16	$m_e^s = (32 - 32) * 2m_Q$	0						
16	$m_e^{sc} = 2(w_{16} - 1) * 2m_Q$	0.5022						
	↑ em charge attached to roton 1 ↑							
32	$m_{\tau}^{s} = (64 + 64) * 4m_{Q}$	1665.46						

Figure 14: Table of skeleton masses of lepton models. The skeleton masses of muon and tauon follow from eqn. 56. The skeleton masses of the electron and the electron neutrino are zero. The addition of an electromagnetic (em) charge to one roton of the electron model generates the main part of the measured electron mass.

Skeleton and dress components of the basic space models of muon, electron, and tauon are depicted in the structural diagrams of Fig. 15.

The dresses of the models show an 'electroweak head' of four rings, at roton 1 fully occupied by the energy factors of nine charges, one em charge and eight charges realizing the capability for weak interaction.

The fluctuating charge pairs are symmetrically distributed between roton 1 and 2, identically structured in all cases.

In the electron model, the energy factors  $2a_k$  and  $a_k^2$  of fluctuating charges are present. In the electroweak head, one has to assume  $a_{32}^2$  instead of  $2a_{32}$  as energy factors of the weak charged rings. These assumptions are dictated by the experimental values for the mass and the anomalous magnetic moment of the electron, see the Figs. 16 and 17 later on.

The dress contribution  $w_k$  of the em charge plays a special role because of its separate energy conservation, see equation (48). Consequently, the dresses in the lepton mass formulas are different for basic space and space-time:

$$M_{lept} = [D_1(k) + D_2(k)] * \frac{k}{8} M_Q(k) \quad in \ basic \ space$$
 (60)

$$m_{lept} = [D_1(k) + \Delta D_1(k) + D_2(k)] * \frac{k}{8} m_Q(k) \quad in \ space - time$$
 (61)

$$\Delta D_1(k) = w_k - 1 \tag{62}$$

The contribution of the self-energy of the em charge appears twice in the space-time formula,  $w_k - 1$  is contained in  $D_1(k)$  as well in  $\Delta D_1(k)$ . This reflects

Muon								Ta	uon		
↑ ro	ton 1			↓ roton 2		↑↑ rc	↑↑ roton 1		↓ roton 2		
charges	dress	skel.	skel.	dress	charges	charges	dress	skel.	skel.	dress	charges
θ	W8	$M_Q$	$M_Q$	1		Θ	W32	$4M_Q$	$4M_Q$	1	
⊕ ⊕	a <sub>16</sub>	٠	•	1		⊕ ⊕	a <sub>64</sub>	4 •	4 •	1	
9 9	a <sub>16</sub>		•	1		9 9	a <sub>64</sub>	4.	4 •	1	
$\oplus \ominus \oplus \ominus$	<i>u</i> <sub>32</sub>	•	•	1		$\oplus \ominus \oplus \ominus$	u <sub>128</sub>	4.	4 •	1	
$2 \times \oplus \wedge \ominus$	4b8	•	•	4b8	$2 \times \oplus \wedge \ominus$		12	4 •	4 •	12	
$2 \times \oplus \wedge \ominus$	4b8		•	4b8	$2 \times \oplus \land \ominus$	$20 \times \oplus \land \ominus$	$40b_{32}$	4.	4 •	$40b_{32}$	$20 \times \oplus \land \ominus$
$2 \times \oplus \land \ominus$	4b8		•	4b8	$2 \times \oplus \land \ominus$	$8 \times \oplus \land \ominus$	$8b_{32}^2$	4.	4 •	$8b_{32}^2$	$8 \times \oplus \land \ominus$
16	16 rings per roton, ring mass $M_Q$					64	rings p	er roto	n, ring 1	mass 4	$M_Q$

Electron						
↑↑ ro	oton 1			↓ (anti) roton 2		
charges	dress	skel.	skel.	dress	charges	
Θ	W16	$2M_Q$	$-2M_Q$	1		
$\oplus \oplus \land \ominus \ominus$	$a_{32}^2$	2 •	<b>−2</b> •	1		
	1	2 •	<b>−2</b> •	1		
⊕⊖⊕⊖	u <sub>64</sub>	2 •	<b>−2</b> •	1		
	4	2 •	<b>−2</b> •	4		
8 × ⊕ ∧ ⊖	$8a_{16}^2$	2 •	<b>−2</b> •	$8a_{16}^2$	8×⊕∧⊖	
$8 \times \oplus \land \ominus$	16a <sub>16</sub>	2 •	−2 •	$15a_{16} + b_{16}$	8×⊕∧⊖	
32 r	ings per	roton,	ring ma	ss $2M_Q$ or $-2$	$M_{\mathcal{Q}}$	

Figure 15: Models of muon. electron, and tauon as structural diagrams. The dot  $\bullet$  symbolizes one mass quantum  $M_Q$  in basic space. The diagrams show the left chiral variant, electroweak charges are fixed to roton 1. They would change to roton 2 at the right chiral variant.

the fact, that the rotating mass is halved during the transition from basic space to space-time, whereas the mass equivalent of the contribution of a rotating em charge is not halved, due to the energy conservation of its self-energy, see eqns. (47) and (48).

The formulas for lepton masses  $m_{lept}$  in space-time are given in the table Fig. 16. In contrast to the structural diagrams, which refer to basic space models, the formulas of  $m_{lept}$  are valid in space-time, they contain  $2w_k$  instead of  $w_k$ . The last column in Fig. 16 shows the deviation of the model values from the measured values taken from [7].

 $m_{lept}(k) = (D_1(k) + \Delta D_1(k) \pm D_2(k)) * \frac{k}{2} m_O$ ; minus sign for the electron

lepton	$m_{lept,\text{mod}} (MeV/c^2)$ ; $m_Q = 3.252848 \ MeV/c^2$	dev.
μ, k=8	$(2w_8 + 2a_{16} + u_{32} + 24b_8 + 3)m_Q = 105.657817$	$5.3 * 10^{-6}$
e, k=16	$(2w_{16} + a_{32}^2 + u_{64} - 4 + a_{16} - b_{16}) * 2m_Q = 0.510945$	$1.1 * 10^{-4}$
e pure em	$m_e \approx (2w_{16} - 2) * 2m_Q = \frac{32\alpha/\pi}{1 - 16\alpha/\pi} * 2m_Q = 0.50224$	$1.7 * 10^{-2}$
τ , k=32	$(2w_{32} + 2a_{64} + u_{128} + 16b_{32}^2 + 80b_{32} + 27) * 4m_Q = 1776.94$	$3.3 * 10^{-6}$

Figure 16: Mass formulas for lepton models. The electron mass is calculated as difference between the masses of roton 1 and 2 as shown in the structural diagram. The main part of this difference comes from the electromagnetic charge, see the row 'e pure em'.

The deviation of the model value of the muon mass could be reduced down to  $3.2*10^{-8}$  by a small correction to the self-energy of one fluctuating charge, one obtains:

$$m_{\mu} = (2w_8 + 2a_{16} + u_{32} + 23.5b_8 + 0.5d_8 + 3)m_Q = 105.6583789 \text{ MeV/}c^2.$$

We remark, that the experimental value of  $m_{\mu}$  is used as one input for the derivation of the mass quantum  $m_Q$ , see eqn. 69.

# 7.2 Gyromagnetic properties and the anomalous magnetic moment

The angular momenta (spin contributions) can be calculated separately for the two rotons of a biroton, see Fig. 2 and eqn. (8). The angular momenta are conserved, that means identical in basic space and space-time:

$$\left|\overrightarrow{L_1}\right| = r_1 M_1 c = \hbar \; ; \quad \left|\overrightarrow{L}\right| = \left|\overrightarrow{L_1} + \overrightarrow{L_2}\right| = \frac{1}{2}\hbar ; \quad conserved$$
 (63)

The same is true for the magnetic momenta. We assume, that the em charge is located at one of the both rotons - at roton 1 in the left chiral case and at roton 2 for right chiral models. If roton 1 carries the em charge, also the magnetic

moment  $\mu_1$  is conserved:

$$|\overrightarrow{\mu_1}| = \frac{1}{2}q_1cr_1 = \frac{q_1}{2M_1} |\overrightarrow{L_1}| = \frac{q_1}{4m_1} |\overrightarrow{L_1}|; \quad conserved$$
 (64)

The gyromagnetic g-factor  $g_1$  of the first roton is defined by equation (65) in basic space and by (66) in space-time. That means, the g-factor  $g_1$  is not conserved during the transition. The g-factor of the second roton is zero,  $g_2 = 0$ .

$$g_1 = \frac{|\overrightarrow{\mu_1}|}{\frac{q_1}{2M_1}|\overrightarrow{L_1}|} = 1; \quad not \ conserved$$
 (65)

$$g_1 = \frac{|\overrightarrow{\mu_1}|}{\frac{q_1}{4(m_1 + \Delta m_1)} \left| \overrightarrow{L_1} \right|} = \frac{m_1 + \Delta m_1}{m_1} \tag{66}$$

We replace  $\left|\overrightarrow{L_1}\right| = \hbar$  by  $\left|\overrightarrow{L}\right| = \frac{\hbar}{2}$  and obtain the gyromagnetic g-factor  $g_{lept}$  of the biroton. The result is  $g_{lept} \approx 2$ , approximately the result  $g_{lept} = 2$  of the Dirac-theory. The fraction on the right side of (66) can be reduced by the mass quantum  $m_Q$ . Using eqn. (61) we get a formula with dresses instead of masses:

$$g_{lept} = \frac{|\overrightarrow{\mu_1}|}{\frac{q_1}{4(m_1 + \Delta m_1)} |\overrightarrow{L}|} = 2 \frac{m_1 + \Delta m_1}{m_1} = 2 \frac{D_1 + \Delta D_1}{D_1}$$
(67)

$$a_{lept} = \frac{g_{lept} - 2}{2} = g_1 - 1 = \frac{\Delta D_1}{D_1} \tag{68}$$

This formula for the anomalous magnetic moment  $a_{lept}$  can be tested for charged leptons using  $\Delta D_1(k) = w_k - 1$  and the expressions for  $D_1$  following from the mass formulas (see Fig. 16). The anomalous magnetic moment does not depend on assumptions for the mass quantum and is not influenced by the second roton. This is of particular importance for the model of the electron, where the electrically neutral antiroton is without effect on  $a_e$ .

The roton which carries no em charge, for instance roton 2, has the magnetic moment  $\mu_2 = 0$ .

Numerical values for the anomalous magnetic moment are summarized in Fig. 17.

We remark, that a change of the self-energy of one fluctuating charge in roton 1 of the muon from

$$b_8 = 1 + 8z = 1.018583$$
 to  $(b_8 + d_8)/2 = 1 + 8z + \frac{1}{2}(8z)^2 = 1.018755$ 

would result in a decrease in the deviation of  $a_{\mu}$  from  $1.1*10^{-5}$  down to  $6.5*10^{-8}$ . For the reason of simplicity, we don't use here this correction. The model proposed here cannot achieve the high precision of the exact calculations in QED, however the model establishes a kind of quantum electrodynamics in basic space.

r)		
	$a_{lept}(k) = \frac{g_{lept}(k)-2}{2} = g_1(k) - 1 = \frac{\Delta D_1(k)}{D_1(k)}$	
lepton	$a_{lept}(k)$ ; $\Delta D_1(k) = w_k - 1 = \frac{k\alpha/\pi}{1 - k\alpha/\pi}$	dev.
μ, k=8	$a_{\mu} \approx \frac{w_8 - 1}{w_8 + 2a_{16} + u_{32} + 12b_8} = 1.1659329 \times 10^{-3}$	$1.1 * 10^{-5}$
e, k=16	$a_e \approx \frac{w_{16}-1}{w_{16}+a_{32}^2+u_{64}+8a_{16}^2+16a_{16}+5} = 1.1596525 \times 10^{-3}$	$3 * 10^{-7}$
τ, k=32	$a_{\tau} \approx \frac{w_{32}-1}{w_{32}+2a_{64}+u_{128}+8b_{32}^2+40b_{32}+12} = 1.17601 \times 10^{-3}$	1 * 10 <sup>-3</sup> *)

<sup>\*)</sup> Comparison with SM calculation, exp. value not available

Figure 17: The anomalous magnetic moment of lepton models, The formulas for  $a_{lept}$  depend on the ring parameter k, however they are independent of the mass quantum and of the second roton. Thus the electron shows the same behavior as muon and tauon, despite the negative mass of the second roton.

### 7.3 The derivation of the mass quantum

The mass quantum  $M_Q$  in basic space and  $m_Q = \frac{1}{2}M_Q$  in space-time has been introduced already as the ring mass of the muon in a generalized form, see section 7.1. We use experimental values of the muon mass  $m_{\mu}$  and the muon anomalous magnetic momentum  $a_{\mu}$ , taken from [7] and [21] as input, inorder to derive the precise numerical value of the mass quantum.

$$m_{\mu} = 105.6583755(23) \ MeV/c^2$$
  
 $a_{\mu} = 1.16592059(22) * 10^{-3}$ 

We use the equations (61) and (68) and obtain the dress contributions

$$D_1^{opt}(8) = \frac{w_8 - 1}{a_\mu} = \frac{8z}{1 - 8z} * \frac{1}{a_\mu} = 16.2398756$$
;  $(z = \alpha/\pi)$ 

$$\Delta D_1(8) = w_8 - 1 = \frac{8z}{1 - 8z} = 1.893441 \times 10^{-2}$$

$$D_2(8) = 12b_8 + 4 = 12(1+8z) + 4 = 16.2229907$$

The term  $D_1^{opt}(8)$  represents a numerical dress value which would exactly reproduce the anomalous magnetic moment of the muon, independent of a more or less precise realization by a model expression. The muon resulting from this mass becomes

$$m_{\mu} = (D_1^{opt}(8) + \Delta D_1(8) + D_2(8)) * m_Q.$$

Using this expression, the mass quanta in space-time  $m_Q$  and in basic space  $M_Q$  get the values

$$m_Q = m_\mu / ((w_8 - 1)(\frac{1}{a_\mu} + 1) + 12b_8 + 4) = 3.2528485 MeV/c^2$$
 (69)

$$M_Q = 2m_Q = 6.505697 \ MeV/c^2 = 1.159746 \times 10^{-26}g$$
 (70)

Using the model structure of the muon shown in Fig. 15, one obtains a value for  $m_Q$  higher by  $5*10^{-6}$ . This deviation goes down to  $3*10^{-8}$  by adapting for one fluctuating charge the correction

$$b_8 = 1 + 8z \rightarrow \frac{1}{2}(b_8 + d_8) = 1 + 8z + \frac{1}{2}(8z)^2; \quad z = \frac{\alpha}{\pi}$$

This correction would approximate the optimum dress value  $D_1^{opt}(8)$  much better and also result in a more precise value for the anomalous magnetic moment, see section 7.2.

We estimate the systematic error of the mass quantum to be of the order  $10^{-6}$ .

### 7.4 The weak interaction

#### 7.4.1 The weak coupling strength

The effective weak coupling strength  $\alpha_{w,eff}(k)$  is determined by two doubly charged rings, related to a roton with 2k rings.

$$\alpha_{w,eff}(k) = \frac{2\pi}{2k}(2a_{2k} - 2) = \frac{2\pi}{k}(a_{2k} - 1) = 4\alpha_{em} * a_{2k}$$
 (71)

The expression for  $\alpha_{w,eff}(k)$  is similar to eqn. (53) for the effective electromagnetic coupling strength  $\alpha_{em,eff}$ . The relation

 $\alpha_w \sim g^2$  represents the counterpart to  $\alpha_{em} \sim e^2$ , see eqn. (52). The weak coupling constant g used in the SM corresponds to the elementary charge e.

The 'Weinberg relation'

$$W = \frac{\alpha_{em,eff}}{\alpha_{weak,eff}} \tag{72}$$

between the two effective coupling strengths differs for the two forms of the weak dress contributions: Separated doubly charged rings have the energy factors  $2a_{2k}$ , a condensed ring has the energy factor  $a_{2k}^2$ .

The relation W becomes  $W_s(k)$  for separated or  $W_c(k)$  for condensed weakly charged rings:

$$W_s(k) = \frac{w_k - 1}{2(a_{2k} - 1)} = \frac{1}{4} - \frac{1}{4}kz - \frac{1}{4}k^2z^2 + \frac{7}{4}k^3z^3 + O\left(z^4\right)$$
 (73)

$$W_c(k) = \frac{w_k - 1}{a_{2k}^2 - 1} = \frac{1}{4} - \frac{1}{2}kz - \frac{1}{4}k^2z^2 + 2k^3z^3 + O\left(z^4\right)$$
 (74)

The Table in Fig. 18 contains the relations W(k) of electromagnetic (em) and weak coupling strengths calculated according to two model variants. The green colored possibilities are favored, due to constraints coming from the modelling of the anomalous magnetic momenta. The model value  $W_c(16)$  for the electron approximates the value of  $sin^2\Theta_W$  of the Standard Model.

## 7.4.2 The weak parity violation

The left – right asymmetry between the two states of a biroton (or an antibiroton) represents a geometric picture of the weak parity violation. Doubly charged rings belonging to the weak charge group are depicted in Fig. 19. The

W =	$W = \frac{\alpha_{em,eff}(k)}{\alpha_{weak,eff}(k)} \Leftrightarrow \sin^2 \Theta_W = 0.23129$								
k	lepton	$W_s(k)$	$W_c(k)$						
8	muon	0.24528	0.24064						
16	electron	0.24045	0.23117						
32	tauon	0.23068	0.21222						

Figure 18: The relation W of electromagnetic and weak coupling constants. The green colored fields are preferred due to constraints from other calculations. The value for the electron agrees well with the SM-value of  $\sin^2\Theta_W$ , where  $\Theta_W$  means the Weinberg angle.

two charges of a weak double ring overlap at the right handed variant of the model, not at the left handed; vice versa for anti-birotons (bottom row). The overlap of charges presumably causes the unability of weak coupling.

#### 7.4.3 The weak interaction of different particles

The electroweak theory as part of the Standard Model uses the picture of viirtual heavy gauge bosons, mediating the weak interaction. The measured energy equivalents of the masses of W - and Z - bosons are

 $m_W = 80.379 \pm 0.012$  GeV and  $m_Z = 91.1876 \pm 0.0021$  GeV. The Heisenberg uncertainty relations are used to estimate the time of appearance and the maximum distance passed by the virtual heavy bosons, if they travel with the velocity of light. The result of such an estimation of the travel distance of a virtual  $W^{\pm}$  boson has the form

$$c\Delta t \thickapprox \frac{\hbar c}{2E} = \frac{197.3~(MeV*fm)}{2*80379~(MeV)} = 1.23~\times 10^{-3}~{\rm fm}$$

The very small distance passed by the mediator particle is considered as the 'range' of the weak interaction and as the origin of its weakness.

In basic space, however, the assumption of virtual mediator particles is not useful. Instead, the contact of two circulating structures along a common cycle length is appropriate. The circumference of the biroton determines the contact length in basic space, replacing the propagation length determined by the uncertainty relations in space-time. The cycle lengths of a lepton model has to be multiplied by a small probability in order to get approximately  $10^{-3}$  fm.

The geometry of the biroton is always present, thus one doesn't need virtual particles appearing spontaneously. The two pictures are completely equivalent, and both don't have an experimental verification. The formal description of the two pictures provide identical results. This can be demonstrated on the Fermi-constant, neglecting for a moment the energy dependence of this 'constant'.

Usually the Fermi constant has the energy-related form

$$\sqrt{2}G_F/(\hbar c)^3 = 1.1663787(6) \times 10^{-5} \ [GeV^{-2}]$$

This form is convenient for a theory using the exchange of heavy particles (gauge bosons  $W^{\pm}$  or  $Z^{0}$ ). Instead, we use the length-related form, without

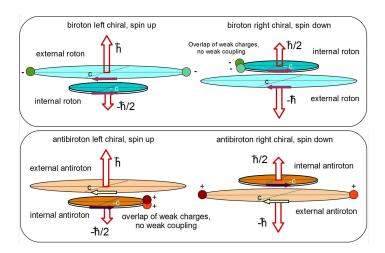


Figure 19: Four states of a biroton or an anti-biroton corresponding to four states of a Dirac spinor in rest frame. The overlap of two weak charges during the  $4\pi$  - cycle of the internal roton disables the capability of weak interaction. This restricts weak coupling to left chiral particles and right chiral antiparticles. The combination of Parity operation and Charge conjugation preserves the capability of weak interaction, corresponding to the diagonal in the Figure from the upper left to the lower right panel.

changing the numerical value:

$$\sqrt{2}G_F/(\hbar c) = (\hbar c)^2 * \sqrt{2}G_F/(\hbar c)^3 = 4.541638 \times 10^{-7} [fm^2]$$
 (75)

The Fermi constant can be expressed in basic space as the effective coupling strength  $\alpha_{weak,eff}$  multiplied by two 'probability lengths'  $L_w(1)$  and  $L_w(2)$ , belonging to one of the two weakly interacting particles. Each value of  $L_w$  is determined by the cycle length  $l_c$  and a product of two probabilities  $p_w$ .

$$\sqrt{2}G_F/(\hbar c) = \alpha_{weak,eff} * L_w(1)L_w(2) \quad [fm^2]$$
(76)

$$L_w = l_c * p_w^2 \approx 762z^2 \ [fm]$$
 (77)

$$l_c = 2\pi r_1 = \frac{h}{M_1 c} = \frac{4hc}{k^2 m_Q c^2} \approx \frac{762}{k^2} [fm]$$
 (78)

$$p_w \approx kz \; ; \qquad z = \frac{\alpha}{\pi}$$
 (79)

The probability of a charged ring to be 'active', that means ready to interact weakly, is assumed to be  $p_w \approx kz$ , such that  $p_w^2$  represents the probability, that two rings of a roton are simultaneously active. The dependences of  $l_c$  and  $p_w^2$  on the ring parameter k cancel each other, see the eqns. (78) and (79). The probability length

$$L_w \approx 762z^2 = 4.11 \times 10^{-3} \ fm$$

becomes independent of k in the approximation used here. The effective weak coupling strength can be taken from eqn (71) also as an approximation independent of k:

$$\alpha_{weak,eff} \approx 4\alpha_{em} \approx 2.92 \times 10^{-2}$$
.

The result of this coarse estimation is a numerical value in the region of the Fermi constant in its length-related form, independent of the type and energy of the two interacting particles:

$$\alpha_{weak,eff} * L_w^2 \approx 2.92 \times 10^{-2} * (762z^2)^2 = 4.94 \times 10^{-7} [fm^2]$$
 (80)

This example shows, that the interpretation of the electroweak theory possibly could be changed without changing the physical content, tat means the observable quantities. Any term of the form

$$\frac{\hbar c~[GeV*fm]}{Mc^2~[GeV]}$$

can be interpreted as describing the exchange of a virtual particle with the mass M (in GeV) in the 'physical vacuum' or alternatively as describing the common circulation along a contact length  $\hbar c/M$  (in fm) within a temporary common eigenspace. Such a change in interpretation would be possible without any change in the formulation of measuring results.

Fig. 20 depicts the weak interaction as a point-like process involving four fermions (upper panel), as an interaction of two 'currents' by exchanging the charged  $W^-$  - boson (second panel) and as a process of common circulation of two interacting particles, forming a temporary common eigenspace (lower panel).

## 8 Mono-rotons and its role in cosmology

### 8.1 Comparison of quanta of charge and mass

The basic space models of the electron and of neutrinos consist of positive as well as negative mass quanta, this enables to explain the small masses of these particles. This assumption corresponds to the existence of elementary charges of both signs. However, positive and negative charges can be observed in spacetime, however masses observed in space-time are always positive. Fig. 21 shows the comparison of attraction or repulsion of interacting charge or mass quanta of different sign. Only the green colored interactions are directly observable in space-time. The positive mass quanta  $M_Q^+$  represent in basic space matter in contrast to antimatter, which consists of negative mass quanta  $M_Q^-$ .

The pairwise internal compensation of charges as well as masses are necessary assumptions for the construction of basic space models of leptons. The internal compensation of fluctuating charge pairs contributes to the self energy, without changing the observable charge state of the particle. The internal compensation of mass quanta in neutrino models allows to explain their weak interaction

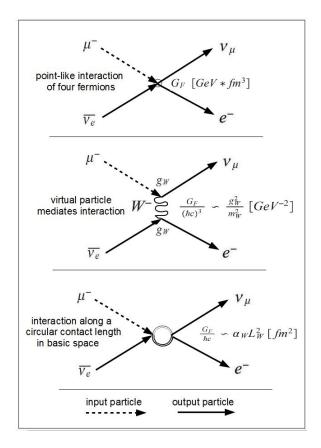


Figure 20: The weak interaction in comparison of three interpretations. The upper panel shows Fermi's four-fermion theory with interaction at a point. The second panel describes the exchange of a virtual  $W^{\pm}$  boson according to the electroweak theory of the Standard Model. The lower panel shows the weak interaction of two particle models by a common contact length in basic space. The weak interaction appears here as the temporary generation of a common eigenspace of the two interacting particles.

capabilities, without appearing as an observable mass of the particle. The internal compensation of matter and antimatter implies the existence of 'mixed matter', a third kind of particles, assumed in models of neutrinos, the electron and positron. We remark, that the 'mixing' does not occur in space-time, where zero or nearly zero particle masses appear as the result of mixing.

Interaction partners	Electrostatics	Gravostatics
equal positive signs	$e^+ \iff e^+ \text{ repulsion}$	$M_{\mathcal{Q}}^{\scriptscriptstyle +}\Rightarrow \Leftarrow M_{\mathcal{Q}}^{\scriptscriptstyle +}$ ; attraction
equal negative signs	$e^- \iff e^- \text{ repulsion}$	$M_{\mathcal{Q}}^{-}\Rightarrow \Leftarrow M_{\mathcal{Q}}^{-}$ ; attraction
opposite signs	$e^+ \Rightarrow \Leftarrow e^-$ attraction	$M_{\mathcal{Q}}^+ \Longleftrightarrow M_{\mathcal{Q}}^-$ repulsion
opp. signs, compensated	$\left\{e^{+} + e^{-}\right\} \equiv 0$	$\{M_Q^+ + M_Q^-\} \equiv 0$
pairwise compensation in	photon; charge pairs	electron; neutrinos

Figure 21: Comparison of interacting quanta of charge and mass. Gravostatics means Newtonian gravity applied to mono-rotons and extended into basic space. The gravitational interactions - attraction and repulsion - are opposite to the electrostatic interactions. Only the green colored interaction types are observable in space-time. Particles with internal compensation of electrostatic effects appear uncharged, particles with internal compensation of gravitational effects appear massless or nearly massless. Masses of antiparticles and mass differences of lepton models become always positive in space-time, thus no anti-gravitation can be observed.

The assumption of negative mass quanta in basic space does not lead to anti-gravitational effects in space-time. Anti-particles have positive masses in space-time. If the difference of roton masses in a lepton model turns out to have a negative sign, so it will appear in space-time as a small positive mass, see Fig.22.

The two rotons have a relative independence in different states of a biroton. This can be seen in the existence of a nonlocal state of a biroton during translation, in the independent development of magnetic properties of a roton carrying an em charge, and in other instances. Therefore it is a suggestive assumption, that rotons may exist also completely isolated, as 'mono-rotons'. Single entities of positive and negative mass live in its eigenstate at a subparticle-level, similar to entities of positive and negative charge, which live at the particle-level. However, only a biroton consisting of two bound rotons can translate in space-time, developing time and spatial coordinates. Thus single rotons are *immobile* and *inobservable* in space-time by direct experiments. A possible means to detect single mono-rotons indirectly could be by its gravitational interaction. Gravity has to be considered as existing already at the subparticle level, whereas the interaction types generated by electromagnetic or weak charges develop only at the particle level.

Biroton	E <sub>rot</sub>	E <sub>rot</sub>	Eo
	per roton	per biroton	in spacetime
roton 1 +M/2 roton 2 +M/2	+Mc <sup>2</sup> /4 +Mc <sup>2</sup> /4	+Mc <sup>2</sup> /2	m₀ c² Muon
roton 1 +M/2 anti-roton 2 -M/2	+Mc <sup>2</sup> /4	≈ 0	≅ 0 Muon- neutrino
anti-roton 1 -M/2 +M/2	-Mc <sup>2</sup> /4 +Mc <sup>2</sup> /4	≈ 0	≷ 0 Muon- antineutrino
anti-roton 1 -M/2	-Mc <sup>2</sup> /4	-Mc <sup>2</sup> /2	r m₀ c² Anti-muon

Figure 22: The biroton as combination of rotons and anti-rotons. The internal combination of rotational energies of the rotons may result in positive, negative or even nearly zero total energies of the biroton in basic space. The invariant mass in space-time is always positive. Negative masses are restricted to circular motion, linear translational motions require positive masses and energies.

## 8.2 The joint emergence of particles and space-time

A certain amount of matter and antimatter, consisting exclusively of single mono-rotons, is considered as a logical starting point of the cosmological development. The first emerging biroton could create an isolated, short-lived piece of space-time, with a linear spatial distance  $x_c$  and a cycle time  $t_c$  as extensions in space-time, see eqns. (6) and (27). One could speak of the first 'quanta of space-time' with dimensions depending on the geometry of the first emerging birotons.

A big number of emerging birotons, accompanied by emerging quanta of space-time could appear and disappear like flashs of lightning. During a short time interval, several isolated pieces of space-time could be grown together, forming a bigger region of space-time, populated by several birotons travelling through space. Such processes could occur many times simultaneously and yield finally a general space-time in the universe. The original quanta of the emerging space-time would loose importance and disappear, not identifiable in the continuum of general space-time.

Presumably the result could be the same as that caused by a 'Big Bang' and a subsequent phase of 'cosmic inflation' according to the standard model of cosmology. The joint emergence of a huge amount of birotons and quanta of space-time would replace the singular 'Big Bang' and the cosmic inflation. Such a development could generate distributed observable matter in a wide space all at once, avoiding the singularities and difficulties connected with the cosmologic standard model.

The joint emergence of particles and space-time does not exclude a further expansion of the universe, but it didn't start at a singular point.

The proposal outlined in this article has to be checked with respect to its compatibility with all observed data such as the cosmic background radiation.

# 8.3 Mono-rotons as candidates of dark matter and dark energy

A second question is the possible current existence of isolated mono-rotons within the general space-time and the possibilities for a proof of the existence of such mono-rotons. The presumed properties of a mono-roton can be estimated from the structure diagrams of lepton models. We remark, that the quantities given in Fig. 23 do not represent particle properties observable in space-time. The indicated tentative values could possibly be changed by a factor of 2, such that  $r \approx 30$  fm and  $|L| = 2\hbar$ . This would give the spin of a graviton, however, gravitons are assumed to be massless in space-time. So in addition one had to assume that gravitons consist of mixed matter like neutrinos, and they should be observable in space-time. In contrast, we emphasize the *inobservability* of mono-rotons.

Quantity	Symbol	Value
rotating energy,	+2M <sub>Q</sub> c <sup>2</sup>	E <sub>rot</sub>   ≈ 13 MeV
minimum	-2M <sub>Q</sub> c <sup>2</sup>	
radius, maximum	r = ħ/2Μ <sub>Q</sub> c	r ≈ 15 fm
angular momentum	$ L  = 2M_Qc*r$	L  = ħ

Figure 23: Hypothetical properties of mono-rotons, inobservable for direct experiments in space-time.

If mono-rotons would exist in the current universe, we could expect gravitational effects without observable masses as an origin. The existence of mono-rotons possibly could explain, why particles realizing the effects of dark matter and dark energy couldn't be discovered yet. It has to be checked, whether positive mono-rotons could represent dark matter, while negative mass quanta represent dark energy. The existence of negative mono-rotons in the current universe would not result in observable anti-gravitation. Isolated mono-rotons are not able to translate in space-time, they are motionless, 'fixed' to its position. Consequently, negative mono-rotons could not stick together, forming heavy stellar antimatter objects. Nevertheless, they could contribute to the expansion of the universe.

The theory of Newtonian gravity can be extended to an application for the interaction of mono-rotons. We call this theory "gravostatics" in analogy to electrostatics. The paralleles between these theories are outlined in Fig. 24. The existence of a mass quantum  $M_Q$ , independent of the gravitational constant  $G_N$ , allows to define a dimensionless coupling strength of gravity  $\alpha_G$  in analogy to the finestructure constant  $\alpha$ , see the last lines in the table Fig. 24. The Planck mass

could not be used for the definion of  $\alpha_G$ , because this would produce a circle definition. The Planck mass itself is defined using the gravitational constant.

$\alpha$ ; $\alpha_G$ constants	Electrostatics	Gravostatics	Charge equivalent
CGS-Gaussian	$\alpha = e^2/\hbar c$	$\alpha_G = G_N M_Q^2 / \hbar c$	$e^{\pm} \leftrightarrow \pm \sqrt{G_N} M_Q$
SI system	$\alpha = k_C * e^2/\hbar c$	$\alpha_G = G_N * M_Q^2/\hbar c$	$e^{\pm}\leftrightarrow M_{Q}^{\pm}$ ; $k_{C}\leftrightarrow G_{N}$
numeric values	$\alpha\approx7.2976\times10^{-3}$	$\alpha_G\approx 2.~839\times 10^{-43}$	
reciprocals	$\frac{1}{\alpha} \approx 137.035999$	$\frac{1}{\alpha_G} \approx 3.5218 \times 10^{42}$	

Figure 24: Comparison of coupling strengths in electrostatics and gravostatics. The mass quantum  $M_Q$  allows the definition of a gravitational coupling strength  $\alpha_G$  and provides a quantization of gravity, restricted to basic space.  $k_C$  means the Coulomb constant,  $G_N$  the Newtonian gravitational constant.

The relation of the dimensionless constants  $\alpha_G$  and  $\alpha$  amounts to

$$\frac{\alpha_G}{\alpha} = 2.839 * 10^{-43} / (7.2976 * 10^{-3}) \approx 3.89 \times 10^{-41}$$
 (81)

The rotating mass quantum  $M_Q$  causes a quantization of gravity, restricted to basic space. This restriction corresponds possibly to recent results of quantum gravity. Quantum gravity (QG) goes in the direction of a non-spatiotemporal theory [28][16], that means, QG assumes a level 'below' the observable properties of space-time. "It turns out that space-time is absent at the most fundamental level and emerges only in an appropriate limit" [18].

The mass quantum  $\pm M_Q$  represents in basic space the counterpart of the elementary charge  $\pm e$ , if one uses SI units. This is appropriate for practical purposes. However, the mass quantum and the elementary charge cannot be expressed in the same units, this prevents a comparison at a fundamental level. Only within the Gaussian cgs system,  $\pm e$  and  $\pm e_G = \pm \sqrt{G_N} * M_Q$  have the same purely mechanical units:

$$1\sqrt{g*cm^3}*s^{-1} = 1statCoulomb = 1Franklin$$

The electrostatic and the gravostatic charge can now be compared:

$$e = 4.803204673 \times 10^{-10} \sqrt{g * cm^3} * s^{-1}$$
 (82)

$$e_G = M_Q \sqrt{G_N} = 2.996165 \times 10^{-30} \sqrt{g * cm^3} * s^{-1}$$
 (83)

$$\frac{e_G}{e} \approx 6.24 \times 10^{-21} \; ; \quad \frac{e_G^2}{e^2} = \frac{\alpha_G}{\alpha} \approx 3.89 \times 10^{-41}$$
 (84)

The quantization of gravity in basic space does not hold in space-time. The self-energy of circulating charges contributes to the particle mass and undergoes also gravitational interaction. However, self-energy is not quantized despite of the quantization of charge. Therefore, gravity in general is not quantized in space-time in a similar way as in basic space.

Einsteinian gravity is beyond the scope of this article. Einstein's theory of gravity describes the interaction of macroscopic matter and space-time in the universe, without reference to intrinsic particle properties and to the quantization of charge and mass. The GRT requires the existence of matter and space-time as a prerequisite. A discussion of its emergence from a deeper level of reality doesn't seem to be promising within GRT. On the other hand, the joint emergence of particles and space time discussed in this paper cannot contribute to the knowledge on the macroscopic interaction of massive cosmic objects and the curved space-time, which are successfully described by Einstein's equations.

## 9 Conclusions

The article describes two extensions of the usual theoretical framework of particle physics.

First, space-time is complemented by a multi-dimensional circular eigenspace, fixed to the structure of a particle. The eigenspace of a particle resembles the space spanned by the body-fixed coordinates of a spinning top or a satellite. The entirety of all eigenspaces, called basic space, establishes a new level below space-time.

Second, the fundamental particles, structureless in space-time, receive a composite structure, spatially extended in basic space. The building stones of the structure called 'rotons' resemble spinning tops. Rotons consist of quanta of mass and charge, circulating with the velocity of light in basic space. Two rotons coupled in a biroton represent the minimum structure of a particle. The existence of rotons establishes a new subparticle level of material structure below the particle level.

Both extensions are in principle compatible with Quantum Mechanics (QM) and Quantum Electro Dynamics (QED), however, with the exception of the mass generation by the Higgs-mechanism.

A particle model acquires its mass from two sources: The rotational energy of circulating mass quanta and the self-energy of circulating elementary charges. Self-energy is defined as Coulomb-energy caused by the circular self-distance of the charges. The self-distance, for instance the circumference of the circular motion, is always non-zero and therefore infinities do not occur.

The introduction of negative mass quanta allows also the modelling of leptons with zero or nearly zero mass.

Gyromagnetic properties can be calculated seperately for the two rotons of a charged biroton, which helps to understand the anomalous magnetic moment of particles.

A biroton shows various properties of a Dirac-particle, such as a  $4\pi$  - cycle and four states corresponding to the states of a bispinor. The circular velocities +c and -c of the two rotons of a biroton equal the eigenvalues belonging to the so called "Zitterbewegung" in the Dirac theory.

The translation of a particle model in space-time appears as an intermittent process, consisting of single linear steps always performed with the velocity of light. A local mode and a nonlocal mode of translation appear, caused by translational steps initiated by one of the rotons or by both in cooperation. The probabilistic description of the intermittent translation leads to the laws of the Special Relativity Theory (SRT).

The probability wave of the QM has possibly its origin in basic space. The quantum-mechanical probability density in space-time can be interpreted as an originally Gaussian (bell-shaped) envelope of a number of stochastic translational steps. The relativistic addition of velocities and the special Lorentz transformation follow naturally from the probabilistic expressions, if one considers two or more particle models in relative translation.

The cosmologic implications of the proposed subparticle level of matter and space-time have been discussed using lepton models only, without the knowledge on basic space - models of mesons, baryons, and photons, although such models were developed already [13]. However, this restriction is acceptable for the given purpose, because the structure of birotons is fundamental also to other particle models and all models consist of rotons at the subparticle level.

The cosmologic standard model assumes a start of the cosmic development from a singularity of infinite energy density with an explosive event called 'Big Bang' and a subsequent expansion of the universe during a fraction of a second, faster than the velocity of light, called 'cosmic inflation'.

The model proposed in this article avoids the assumptions of a Big Bang and of an inflation. The starting point of the development was probably an amount of matter / antimatter without charge and other particle properties and without an existence in space-time, completely located in basic space. Matter and energy had the form of massive mono-rotons, that means single isolated circulating subcomponents of particles. Each development of a particle by coupling of two mono-rotons was connected with the generation of a quantum of space-time, surrounding the particle. A big number of such processes at the same time resulted in an amount of particulate matter, distributed in a region of space-time. A joint emergence of particles and of space-time could be eqivalent to the effect of the 'Big Bang' and of cosmic inflation.

This picture of the cosmic evolution has to be checked by the community of physicists and cosmologists. Such a check would require quite unusual assumptions: A new subparticle level of reality, and a circular extra space complementing space-time, inaccessible to direct experiments.

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