Mass Quantization and the Upper Limit of Particle Mass

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Abstract

A model is proposed of particles living in two spaces, such that spacetime is complemented by an individual circular eigenspace of each particle. Two sources of particle mass are considered to exist in the eigenspace.

The first source consists in the rotational energy of circulating mass quanta, this energy appears as main part of the invariant energy (rest energy) in spacetime. The circulating mass quanta build the 'skeleton' of the particle and contribute 95 to 98 % of the particle mass.

The second source of particle mass comes from the self-energy of circulating elementary charges. The self-energy is calculated as the Coulomb energy in a finite circular self-distance of a charge, such as the circumference of the circular motoin.Different types of circulation produce electromagnetic, weak and fluctuating charges which represent different interaction capabilities and form the 'dress' of the particle model.

Neutrino masses are modelled as the difference between positive and negative mass quanta. The mass quantum is defined using data from the muon and equals approximately $m_Q \approx m_\mu/32 = 3,3 \ \ MeV/c^2$.

The oscillation of neutrinos can be understood by assuming three neutrino types per generation with different circulation radii of the weak charges.

The skeleton masses of mesons and other hadrons consist of mass quanta enhanced by charge clusters of different size. Particles carrying very heavy clusters are not realized in nature. The masses of the Z^0 and the Higgs boson can be modelled only by assuming a split of very heavy clusters into two or three pieces.

This result suggests the conjecture, that particle masses have an upper limit in the region of 400 GeV. The possible consequences on the strategy in high energy physics are discussed. While experiments directed towards the investigation of known particles (neutrinos, Higgs boson) appear as a sensible investment, the perspectives of searches for new particles in the TeV region appear doubtful.

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1 Introduction

The mass of a particle represents its main characteristic property. The scientific description and interpretation of mass developed during the 20. and the first decennials of the 21. century. As the first particles were discovered (1930's) the theory described only massless particles, the input of an explicit mass term would have ruined the consistency of the theory. So it was a big theoretical success, that three groups (Brout, Englert [1], Higgs [2], as well as Guralnik, Hagen, and Kibble [3]) invented 1964 a method, called the BEH - or Higgs-mechanism, how to introduce particle mass of gauge bosons into the theory without an explicit mass term. However, according to this method, mass appeared not fixed to a particle as its own property. In contrast, mass was granted from the environment, by a contact with the Higgs field. This field was assumed to exist everywhere in the universe and to influence the motion of an (originally massless) particle in such a way, as if it would have a 'mass'. This concept of an apparent particle mass became fully accepted after the discovery of the Higgs boson in 2012, the particle belonging to the Higgs field and predicted by Peter Higgs, who received 2013 the Nobel prize in physics together with Francoise Englert.

Despite the theoretical success, the Higgs - mechanism was not able to explain the measured mass value of a certain type of particles and the existence of three particle families. Therefore empirical formulas [20].[21] and several theories without a Higgs field were proposed [4], [5], [10]. Also alternative mechanisms have been investigated [11], [9]. The calculation of selected particle masses was performed using lattice QCD, an established field of research [8], [6], [7]. However, lattice QCD is restricted to particles consisting of quarks and gluons. The origin of the masses of fundamental particles (leptons, quarks) cannot be directly calculated. The three generations (or families) of fundamental particles represent a special problem. The masses of three leptons and six quarks as well as their antiparticles have to be input by hand into the theory of the Standard Model.

The existence of the families shows some kind of self-similarity in particle physics. Because similar self-similarities are found also in other fields, different authors attempted to compare the particle families with fractal structures, with the systematics of crystals or with the planetary system. The introduction of an extra 'familon' particle was also proposed [12]. At present, no accepted explanation for the family structure exists.

In the literature, different variants of mass quantization have been discussed. Several starting points for the definition of a mass quantum have been tested. The range of proposed mass quanta comprises masses as tiny as 2×10^{-65} g [**?**] and goes for astronomical purposes up into the region of the Planck mass [29] .

From an empirical viewpoint, the small mass of the electron seems to be appropriate to construct particles with higher masses. The small mass of the electron has been proposed as mass quantum, because expressions like $2m_e/\alpha \approx$ 140 MeV seem to generate the mass of other particles. Nambu [20] (1952) and Sidhardt [31] (2003) developed such '$\alpha-$ quantized' empirical formulas, and

Varlamov [22] gave 2023 a detailed discussion of this idea and a long list of examples.

Also intermediate values around 70 MeV have been proposed. None of these possibilities was accepted by the physics community. The masses of neutrinos represent a special challenge for the Standard model. The origin of the tiny neutrino masses is unknown and not discussed in connection with mass quantization.

Another problem is the existence of antiparticles, thus one has to discuss a mass quantum together with its counterpart coming from anti-matter.

During the last century, a big number of proposals have been developed to explain the 'intrinsic' properties such as mass, spin and magnetic moment of the electron by a geometric model, in most cases assuming a circular or helical motion. Although such models could explain spin and even the 'Zitterbewegung' of a Dirac-particle, no one of such models was accepted.

The existence of an upper limit of particle mass has been proposed already during the 1990's when preon models were under discussion [30]

The present article describes the mass quantum, already defined for leptons [17], in an application to lepton masses and to the mass scale of mesons up to the top quark. The main features of the model can be summarized as follows:

1. Particles exist as composite and extended entities in a circular extra space, called basic space, and nearly pointlike in space-time ('dual space concept'). So the model does not come in conflict with the size of particles measured in scattering experiments or with Quantum Mechanics.

2. A mass quantum $m_Q \approx \frac{1}{32} m_\mu = 3.3\ MeV/c^2$ is defined in spacetime and twice that value $M_Q = 2m_Q$ as well in basic space. Particle masses smaller than m_Q are modelled as differences between positive and negative partial masses, $\pm\ m_Q$ and $\pm\ M_Q$ can have both signs.

3. The basic-space model of a particle contains a skeleton formed by circulating mass quanta and a dress formed by circulating charges. Only the skeleton mass is quantized, it represents 95 to 98 % of the total particle mass.

4. The skeleton masses of leptons are determined by the number of mass quanta per particle model, which increases $\sim 2^{2n+2}$, where n is the generation number. The model provides an analytical description of the masses of lepton families and allows the calculation of hypothetical masses of a fourth lepton generation.

5. The skeleton masses of hadrons are also determined by the number of mass quanta per particle, the same formation law holds as with lepton families. However, hadron models contain mass quanta enhanced by charge clusters. The product of the energy factors of all charges of a cluster gives a function steeply increasing with the number of charges per cluster, this number depends on the particle generation. The product of the cluster function and the factor 2^{2n+2}, the same as in lepton models, results in an analytical function. This dimensionless function, multiplied with the mass quantum, describes precisely the increasing meson masses in the range from pions to the top quark. This allows the calculation of masses of a fourth particle generation.

6. Circulating charges in basic space are always integer, positive or negative

elementary charges. The relation to the quark model in particular in baryons is beyond the scope of this article, for a discussion see Chapter 2 'Leptons and Quarks' in the book [16].

2 The mass of lepton models

The 'biroton' is intended to represent the model of a Dirac-particle in its rest frame. The biroton lives in basic space and consists of two rotons with opposite spin direction. The two partial spins $\hbar$ and $-\hbar/2$ of the rotons add up to the total spin $\hbar/2$ of the biroton. The two partial masses of the rotons are nearly equal, see Fig. 1.

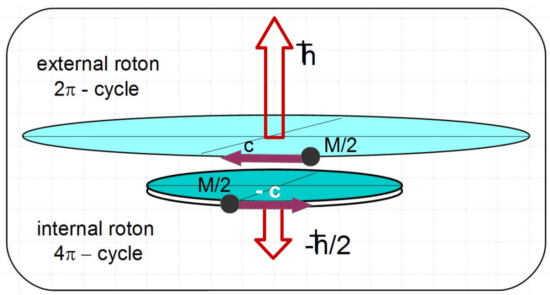

Figure 1: The structure of a biroton, the model of a Dirac - particle in rest frame (left chiral, spin up). The total spin $\hbar/2$ corresponds to the spin of a fermion, in Quantum Mechanics to the spin component s_z in the direction z of the spin axis.

The circular velocities +c and −c of the roton masses correspond to the eigenvalues in the Dirac – theory belonging to the 'Zitterbewegung'. The symmetry of partial masses and the asymmetry of spin components of the rotons reveal some kind of internal supersymmetry between them. The magnitude of the total spin of the biroton $\hbar - \frac{\hbar}{2} = \frac{\hbar}{2}$ corresponds to the quantum number $s = \frac{1}{2}$ of fermions. The quantum number $s+1 = \frac{3}{2}$ would require parallel spins for both rotons, this is not considered to represent a lepton model.

The circulation planes of the rotons represent two separate anticommutative two-dimensional vector spaces. Gamma-matrices serve as unit vectors. The two two-dimensional circulation planes, the common spin axis and the time coordinate result in a total of six dimensions per biroton, see Fig. 2.

The angular momenta of the rotons have a semiclassical definition (all quantities are vectors, we omitted the unit vectors):

$$L_1 = p_{T1} r_1 = \hbar \; ; \quad L_2 = p_{T2} r_2 = -\hbar/2 \tag{1}$$

$$p_{T1} = M_1 c; \qquad p_{T2} = -M_2 c \tag{2}$$

$$r_1 = \hbar/M_1 c; \qquad r_2 = \hbar/2M_2 c \approx r_1/2 \tag{3}$$

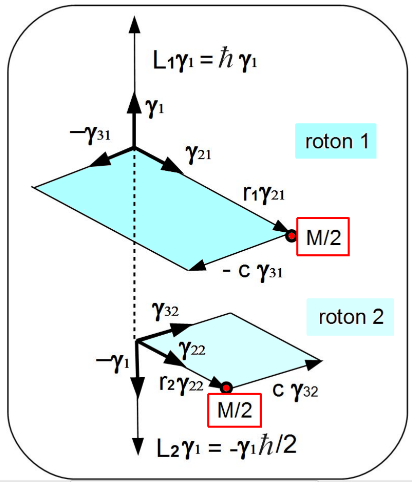

Figure 2: The coordinate schema of a biroton. A two-dimensional circulation plane per roton (colored areas), the common spin axis and the time coordinate result in six dimensions for the biroton. The radius and the tangential momentum of each roton span a noncommutative vector space. The two circulation planes do not belong to the same cylindrical space, because between vectors in $(\gamma_{21}, \gamma_{31})$ and vectors in $(\gamma_{22}, \gamma_{32})$ doesn't exist any mathematical operation.

The rotational energy $\frac{1}{2}Mc^2$ of the circulating mass M is preserved and appears as invariant energy $E_0 = m_0c^2$ in space-time. A kinematic origin of rest mass was proposed also by Buitrago [19].

The cycle length l_c, the circumference of the external roton, represents the (circular) minimum length of the particle model in basic space. The cycle time t_c is the corresponding minimum time interval. A_c represents the cycle action of the biroton, it is equal to Plancks constant h.

$$E_{rot} = \frac{1}{2}Mc^2 = m_0c^2 = E_0 \tag{4}$$

$$l_c = 2\pi r_1 = \frac{h}{M_1c} \approx \frac{hc}{m_0c^2} = \frac{hc}{E_0} \tag{5}$$

$$t_c = \frac{l_c}{c} \approx \frac{h}{E_0}; \qquad A_c = E_0t_c = h \tag{6}$$

The conservation law for the energy (4) implies a halving of the circulating mass M:

$$\frac{1}{2}M \approx M_1 \approx M_2 \approx m_0 \tag{7}$$

Slight differences between the roton masses M_1 and M_2 due to differences of the self-energies of circulating charges are neglected in this formula.

2.1 Types of charged rings

Single electromagnetic charges, double charges of equal sign and neutral charge pairs form different types of rings within a roton. This provides interaction capabilities of the particles and the binding between rings within a particle.

Three types of rings are included into the dresses.

A ring with one single charge provides the capability of electromagnetic interaction,

two doubly charged rings, one charged with 2e+ and one with 2e-, have the capability of weak interaction,

one fourfold charged ring with alternating signs of the charges compensates the high self-energy of both weak rings,

rings with fluctuating neutral charge pairs mediate the binding of the rotons and of different rings within a particle.

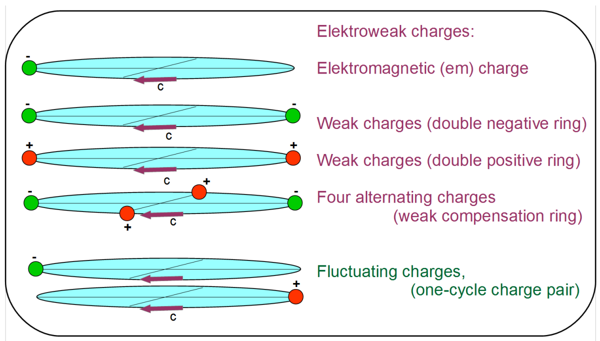

Figure 3: Types of charged rings as parts of the circulating rings of a roton. The charges, symbolized as colored circles, are attached to circulating mass quanta (not shown). Pairs of fluctuating charges remain only one to three cycles at a certain ring and then jump to another ring.

The self-energy of a charge is determined by its self-distance on a circle, that is the circumference (2π-cycle) or twice that value (4π-cycle). The self-energy contributes to the rotational energy of the particle model and therefore to the particle mass.

An additional type of rings contains alternating positive and negative circulating charges, resulting in a *reduction* of the total rotational energy compared to thr uncharged mass quanta. The effect of such a ring is to compensate – at least in part - the big self-energy of doubly charged weak rings.

The Coulomb energy E_C in the distance $2\pi r_1$ from a charge

$$E_C = \frac{\alpha \hbar c}{2\pi r_1} = \frac{\alpha}{2\pi} M_1 c \tag{8}$$

becomes the 'self-energy' of a charge circulating together with the mass M_1 of the first roton. The distance (for instance $2\pi r_1$) becomes the circumference of a circle or another arc length, instead of a linear distance in classical electrostatics. The self-energy in a circular space is always finite, the definition avoids singularities.

The roton is divided into 2k parts, called 'rings', and the charge may be attached to the ring mass $M_r = M_1/2k$ of one ring only, where k is the ring parameter. One gets the self-energy E_{Cr} of the charged ring during a one-cycle circulation

$$E_{Cr} = \frac{k\alpha}{\pi} * \frac{1}{2k} M_1 c^2 = kz * M_r c^2 \tag{9}$$

$$z = \frac{\alpha}{\pi} = 2.322\,819\,465 \times 10^{-3} \tag{10}$$

The self-energy of one circulating charge depends on the number of cycles of continued self-interaction. The possible energy factors enhancing the ring mass are

$b_k = 1 + kz$ (one cycle)
$d_k = 1 + kz + (kz)^2$ (two cycles)
$a_k = 1 + kz + (kz)^2 + (kz)^3$ (three cycles)
$w_k = 1 + kz + (kz)^2 + (kz)^3 + (kz)^4 + (kz)^5 + ...$
$= \Sigma_{i=0}^{i=\infty} (kz)^i = 1/(1 - kz)$ (an infinite number of cycles).

The energy factor of the fourfold ring with alternating charges is $u_k < 1$, it *reduces* the rotational energy of the circulating masses:

$u_k = 1 - kz - (kz)^2 + (kz)^3 + (kz)^4 - (kz)^5 - (kz)^6 + (kz)^7 + ...$

We use the abbreviation $z = \alpha/\pi$.

The charges performimg one, two ot three cycles of continued self-interaction are assumed to jump after a number of cycles to another ring, starting a new self-interaction. This represents the mechanism of binding the rings of a parton and stabilizing it.

The electroweak charges do not jump, they circulate an infinite number of cycles at the same ring an have therefore the energy factor w_k. The self-energy of an em-charge produced in basic space is conserved in spacetime. This is indicated by a factor of one in basic space with $M_r c^2$ and a factor of two in space-time with $m_r c^2$, compare the law of energy conservation (4):

$$E_{Cr}(em) = (w_k - 1) M_r c^2 = 2(w_k - 1) m_r c^2 \tag{11}$$

In contrast to an em charge, weak and fluctuating charges are fixed to a mass quantum such that the self-energy becomes a part of the rotational energy and is conserved just as rotational energy. So the mass equivalent of the self-energy of a fluctuating charge with energy factor b_k appears halved in spacetime together with the mass quanta:

$$\frac{1}{2} b_k M_r c^2 + (w_k - 1) M_r c^2 \Longrightarrow b_k m_r c^2 + 2(w_k - 1) m_r c^2 \tag{12}$$

That means, the rotating ring mass $b_k M_r$ carrying a fluctuating charge becomes the invariant mass $b_k m_r$ in spacetime and the mass equivalent of the self-energy $\sim (b_k - 1) M_r$ becomes halved together with the mass M_r.

2.2 The mass quantum

The biroton represents a model of a lepton. The internal structure consists of a "skeleton", containing circulating mass quanta, and a "dress", containing circulating charges, attached to the mass quanta. A mass quantum m_Q represents the building stone of the skeleton mass. The numerical value of m_Q is determined by using the model formulas for the mass and the anomalous magnetic moment of the muon and constructing a consistent model of both quantities in agreement with experiment [17].

We used the experimental data from the Particle Data Group [32]

$m_\mu = 105.6583755(23)\ MeV/c^2$; relative dev. 2.2×10^{-8}

$a_\mu = 1.16592059(22) \times 10^{-3}$; relative dev. 1.9×10^{-7}

$\alpha = 7.2973525693(11) \times 10^{-3}$: relative dev. 1.5×10^{-10}

$z = \alpha/\pi = 2.322\,819\,465 \times 10^{-3}$

The model-dependent formulas contain symbols for the dresses $D_i(k)$ of the roton $i = 1; 2$ and the muon ring parameter $k = 8$:

$m_{\mu,\mathrm{mod}} = (D_1(8) + \Delta D_1(8) + D_2(8))m_Q$;

$\Delta D_1(8) = (w_8 - 1)$ represents one half of the contribution of an em-charge at $k = 8$. The other half is contained in $D_1(8)$.

$a_{\mu,\mathrm{mod}} = \Delta D_1(8)/D_1(8)$, therefore we set

$D_1(8) = (w_8 - 1)/a_\mu$. ; where $w_8 - 1 = \frac{8z}{1-8z}$

The dresses have per roton 16 places (mass quanta as carrier of circulating charges), where charges can be attached. Four places are reserved in D_1 for the electroweak group $w_8 + 2w_{16} + u_{32}$, 12 places are assumed to be occupied by one-cycle fluctuating charges. Thus we set for the second roton, where 4 places are empty in the left chiral model variant

$D_2(8) = 4 + 12b_8$; $b_8 = 1 + 8z$

Finally we obtain the mass quantum

$m_Q = m_\mu/(D_1(8) + \Delta D_1(8) + D_2(8))$

$= m_\mu/(\frac{8z}{1-8z}(\frac{1}{a_\mu} + 1) + 12(1 + 8z) + 4)$

$$m_Q = 3.252\,848\,46 \quad MeV/c^2 \tag{13}$$

The relative uncertainty of m_Q caused by the input of experimetal data is about 2×10^{-7}. The uncertainty introduced by model assumptions is not exactly known and is estimated to be $\leq 5 * 10^{-6}$. This estimation represents the most conservative interpretation of the small corrections, which appear during the construction of a consistent muon model, see the next section.

2.3 Lepton models and the missing fourth lepton generation

Several particles have masses equal or nearly equal to zero, such as neutrinos. Such masses can be modelled by assuming positive as well as negative mass quanta. We refer in this section to the lepton model introduced in a previous article [17, 35].

2.3.1 Muon and muon neutrinos

The leptons of generation number n = 1 are the muon, muon neutrinos and their anti-particles.

The structure diagram of the charged lepton of the first generation is shown in Fig. 4

Muon					
$\upuparrows$ roton 1			$\downarrow$ roton 2		
charges	dress	skel.	skel.	dress	charges
$\ominus$	w_8	M_Q	M_Q	1	$\square$
$\oplus\ \oplus$	w_{16}	M_Q	M_Q	1	$\square$
$\ominus\ \ominus$	w_{16}	M_Q	M_Q	1	$\square$
$\oplus\ominus\oplus\ominus$	u_{32}	M_Q	M_Q	1	$\square$
$2\times\oplus\wedge\ominus$	$4b_8$	M_Q	M_Q	$4b_8$	$2\times\oplus\wedge\ominus$
$2\times\oplus\wedge\ominus$	$4b_8$	M_Q	M_Q	$4b_8$	$2\times\oplus\wedge\ominus$
$2\times\oplus\wedge\ominus$	$4b_8+\frac{31}{64}(d_8-b_8)$	M_Q	M_Q	$4b_8$	$2\times\oplus\wedge\ominus$
16 rings per roton, ring mass M_Q					

Figure 4: The structural diagram of the muon. One of the four states of a Dirac - particle is shown: Particle with positive energies, spin up. One mass quantum per ring and 16 rings per roton are the main characteristics of the muon model.

The dress components of the muon model are

$$D_1(8) = w_8 + 2w_{16} + u_{32} + 12b_8 + \tfrac{31}{64}(d_8 - b_8) = 16.239\,875 \quad ;$$
$$\Delta D_1(8) = w_8 - 1 = 1.893\,440 \times 10^{-2}$$
$$D_2(8) = 12b_8 + 4 = 16.222\,991$$

These components provide the model representations of mass and anomalous magnetic moment of the muon:

$$\begin{aligned} m_{\mu,\mathrm{mod}} &= [D_1(8) + \Delta D_1(8) + D_2(8)]\, m_Q \\ &= (2w_8 + 2w_{16} + u_{32} + 24b_8 + \tfrac{31}{64}(d_8 - b_8) + 3)m_Q \\ &= 32.481\,800\,32 m_Q = 105.658\,374\,3 \text{ MeV}; \end{aligned}$$
$$a_{\mu,\mathrm{mod}} = \frac{\Delta D_1(8)}{D_1(8)} = \frac{w_8 - 1}{w_8 + 2w_{16} + u_{32} + 12b_8 + \frac{31}{64}(d_8 - b_8)} = 1.165\,920\,617 \times 10^{-3}$$

The number of fluctuating charges is $N_{fluct} = 24$, each roton carries 12 fluctuating charges. That are mainly one-cycle charges (energy factor b_8), with a small probability to change to two cycles (energy factor d_8). The small dress correction $\frac{31}{64}(d_8 - b_8) = 1.672\,6 \times 10^{-4}$ is derived empirically and it improves the approximation of the model from $5.2{*}10^{-6}$ (without correction) to $1.1{*}10^{-8}$ deviation from the measured mass value. A more precise correction would be

$\frac{61}{128}(a_8 - b_8) = 1.676\,2 \times 10^{-4}$ which reproduces the experimental values m_μ and a_μ within 10^{-9}. This confirms the the intention of the semi-empirical definition of the mass quantum. It was designed as to enable a consistent model of the muon.

The series expansion of the dress contributions yields the approximation formula for the muon mass ($z = \frac{\alpha}{\pi}$)

$$m_{\mu,\mathrm{mod}} \approx (32 + 26(8z) - 6(8z)^2 + 113(8z)^3)m_Q = 105.658\,372\ MeV,$$
$$\text{the deviation is } 3.1 * 10^{-8}.$$

The anomalous magnetic moment of the model reaches a deviation of 10^{-5} without and of $2.3 * 10^{-8}$ with the correction term. One obtains

$$a_{\mu,\mathrm{mod}} = \frac{w_8 - 1}{w_8 + 2w_{16} + u_{32} + 12b_8} = 1.165\,932\,625 \times 10^{-3},$$
$$\text{rel. dev. } 10^{-5} \text{ (without correction) or}$$
$$a_{\mu,\mathrm{mod}} = \frac{w_8 - 1}{w_8 + 2w_{16} + u_{32} + 12b_8 + \frac{31}{64}(d_8 - b_8)} = 1.165\,920\,617 \times 10^{-3},$$
$$\text{rel. dev. } 2.3 * 10^{-8} \text{ (with correction)}$$

We get the expected result: The definition of the mass quantum enables a reproduction of the muon data which were used as input.

The neutrino physics has currently a few questions open, in particular the origin and the absolute value of neutrino masses [26]. The muon model has three neutrino variants, see Fig. 5. Each of the three neutrinos has 16 positive mass quanta per roton and 16 negative mass quanta per anti-roton.

The lack of an electromagnetic charge enables the mass quanta to form heavy ring masses at a small number of rings or light ring masses at a higher ring number, see Fig. 5. The weak charge group has to adapt to the different circulation radii and imitates the weak coupling of electron and tauon. The mass eigenstate of a muon-like roton combines with three different flavor eigenstates. This is interpreted as an oscillation between different generations of leptons and as a violation of the lepton number. There is no explanation within the Standard Model for the origin of the neutrino mass and the apparent violation of the lepton number.

The model proposed in this article contains three different distributions of a constant number N_Q of mass quanta per roton between ring masses M_r (in basic space) and ring numbers N_r. These parameters vary during the oscillation between $M_r = M_Q$; $N_r = 16$ (left panel) and $M_r = 4M_Q$; $N_r = 4$ (right panel). The number of mass quanta per roton remains constant $N_Q = 16$. The sum of the skeleton masses of roton and antiroton is always exact zero. The dress changes its circulation radius according to the ring masses, thus the energy factors of the weak rings vary between w_{16} (left panel, corresponding to the original weak coupling capacity of the muon) and w_{64} (right panel, correponding to the weak coupling of the tauon). The dress becomes somewhat asymmetric between roton and antiroton because of the weak charges located only at the roton or at the antiroton. This dress-asymmetry causes a slight mass difference between roton and antiroton and thus a tiny nonzero mass of the neutrino. However, a symmetrization between the dresses of roton and antiroton may take place, which is not displayed in the Figure. Symmetrization may be realized by changing fluctuating charges from one cyle to two or three cycles per self-

⇈ roton 1		↓ antiroton 2	
$D_1^0(8,\mu)$	skel.	skel.	$D_2^0(8,\mu)$
1	M_Q	$-M_Q$	1
w_{16}	M_Q	$-M_Q$	1
w_{16}	M_Q	$-M_Q$	1
u_{32}	M_Q	$-M_Q$	1
$4b_8$	M_Q	$-M_Q$	$4b_8$
$4b_8$	M_Q	$-M_Q$	$4b_8$
$4b_8$	M_Q	$-M_Q$	$4b_8$

⇈ roton 1		↓ antiroton 2	
$D_1^0(16,\mu)$	skel.	skel.	$D_2^0(16,\mu)$
1	$2M_Q$	$-2M_Q$	1
w_{32}	$2M_Q$	$-2M_Q$	1
w_{32}	$2M_Q$	$-2M_Q$	1
u_{64}	$2M_Q$	$-2M_Q$	1
$4b_{16}$	$2M_Q$	$-2M_Q$	$4b_{16}$

⇈ roton 1		↓ antiroton 2	
$D_1^0(32,\mu)$	skel.	skel.	$D_2^0(32,\mu)$
1	$4M_Q$	$-4M_Q$	1
w_{64}	$4M_Q$	$-4M_Q$	1
w_{64}	$4M_Q$	$-4M_Q$	1
u_{128}	$4M_Q$	$-4M_Q$	1

Figure 5: Structural diagrams of three muon neutrinos. The number of mass quanta per roton equals 16 in all cases. This number is realized having a ring mass of $\pm M_Q$ in 16 rings (left panel, corresponding to the muon model), $\pm 2M_Q$ in 8 rings (panel in the middle), or $\pm 4M_Q$ in 4 rings (right panel). The neutrino oscillates between these states, because the skeleton mass is exactly zero and the total energy is nearly zero in all cases. The effects of dress symmetrization, which partly compensate the asymmetry caused by weak charges, are not displayed.

interaction. This would change the enrgy factors of the charges from b_k (one cycle) to d_k (two cycles) or a_k (three cycles).

As an example, the energy contribution of the weak group $2w_{16} + u_{32} - 3$ could be compensated by changes from b_8 to d_8 according to

$$2w_{16} + u_{32} - 3 + 6(d_8 - b_8) = -1.452 \times 10^{-4}$$
$$2w_{16} + u_{32} - 3 + 6.4(d_8 - b_8) = -7.09 \times 10^{-6}$$
$$2w_{16} + u_{32} - 3 + 7(d_8 - b_8) = 2.00 \times 10^{-4}$$

Dress differences in the region 10^{-6} lead to neutrino masses of e few eV. The model allows effects of symmetrization far below these levels. In each case, the neutrino mass would not be a constant value and changes around zero during oscillation. We assume, that the neutrinos are Dirac particles similar to charged leptons and have in basic space the geometry of a biroton (Fig. 1). However, one of the rotons has to be replaced by an anti-roton.

2.3.2 Electron and electron neutrinos

The leptons of generation number n = 2 are the electron, electron neutrinos and their anti-particles.

The number of fluctuating charges is $N_{fluct} = 64$, each (anti) - roton carries 32 fluctuating charges. That are two-cycle and three-cycle charges with the energy factors d_{16} and a_{16}. The dress components of the electron model are

$$D_1(16) = w_{16} + w_{32}^2 + u_{64} + 15a_{16} + d_{16} + 8a_{16}^2 + 5 = 33.285\,570$$
$$\Delta D_1(16) = w_{16} - 1 = 3.859\,967 \times 10^{-2}$$
$$D_2(16) = 5a_{16} + 11d_{16} + 8d_{16}^2 + 8 = 33.245\,627$$

These components provide the model representations of mass and anomalous magnetic moment of the electron. The model of the mass becomes

Electron					
⇈ roton 1			↓ (anti) roton 2		
charges	dress	skel.	skel.	dress	charges
⊖	w_{16}	$2M_Q$	$-2M_Q$	1	□
⊕ ⊕ ∧ ⊖ ⊖	w_{32}^2	$2M_Q$	$-2M_Q$	1	□
□	1	$2M_Q$	$-2M_Q$	1	□
⊕ ⊖ ⊕ ⊖	u_{64}	$2M_Q$	$-2M_Q$	1	□
□	4	$2M_Q$	$-2M_Q$	4	□
8 × ⊕ ∧ ⊖	$8a_{16}^2$	$2M_Q$	$-2M_Q$	$8d_{16}^2$	8 × ⊕ ∧ ⊖
8 × ⊕ ∧ ⊖	$15d_{16} + a_{16}$	$2M_Q$	$-2M_Q$	$5a_{16} + 11d_{16}$	8 × ⊕ ∧ ⊖
32 rings per roton, ring mass $2M_Q$ or $-2M_Q$					

Figure 6: The structural diagram of the electron. One of the four states of a Dirac - particle is shown: Positive energies, spin up, Two mass quanta per ring and 32 rings per roton are the main characteristics of the electron model.

$$
\begin{aligned}
m_{e,\text{mod}} &= [D_1(16) + \Delta D_1(16) - D_2(16)] * 2m_Q \\
&= \left[2w_{16} + w_{32}^2 + u_{64} - 4 + 8(a_{16}^2 - d_{16}^2) + 10(a_{16} - d_{16})\right] * 2m_Q \\
&= 7.854\,315\,012 \times 10^{-2} * 2m_Q = 0.510\,977\,9 \text{ MeV}
\end{aligned}
$$

The model value is about $4.7 * 10^{-5}$ smaller than the measured value. The main contribution comes from the em charge, it amounts to $m_{e,\text{mod}}(em) = (2w_{16} - 2) * 2m_Q = 0.502\,236$ MeV, more than 98 % of the full electron mass. A series expansion of the dress difference in $16z = 16\frac{\alpha}{\pi}$ gives an approximation to the exact value:

$$
m_{e,\text{mod}} \approx (2(16z) - 2(16z)^2 + 136(16z)^3) * 2m_Q = 0.511\,017 \;\; MeV \;; \quad \text{the deviation is } 3.6 * 10^{-5}.
$$

The model of the anomalous magnetic moment is given by

$$
\begin{aligned}
a_{e,\text{mod}} &= \Delta D_1(16)/D_1(16) \\
&= (w_{16} - 1)/(w_{16} + w_{32}^2 + u_{64} + 15a_{16} + d_{16} + 8a_{16}^2 + 5) \\
&= 1.159\,651\,760 \times 10^{-3}
\end{aligned}
$$

This result agrees within $3.6 * 10^{-7}$ with the experimental value.

The electron model has three neutrino variants, see Fig. 7. Each of the three neutrinos has 64 mass quanta per roton. The neutrino variant corresponding to the em-charged electron is shown in the middle. The variants with a halved ring mass (left panel) and with a doubled ring mass (right panel) imitate the weak coupling of muons or tauons, respectively. The three neutrino variants have nearly zero mass and can build a transformation ring like the muon neutrinos.

The weak group of circulating charges causes a slight asymmetry $w_{32}^2 + u_{64} - 2 = -2.26 \times 10^{-5}$ which would lead to neutrino masses in the region of 10 eV. By symmtrization of the dresses can it be reduced, we give a few examples:

⇈ roton 1		↓ antiroton 2	
$D_1^0(8,e)$	skel.	skel.	$D_2^0(8,e)$
1	M_Q	$-M_Q$	1
w_{16}^2	M_Q	$-M_Q$	1
1	M_Q	$-M_Q$	1
u_{32}	M_Q	$-M_Q$	1
12	M_Q	$-M_Q$	12
$16d_8^2$	M_Q	$-M_Q$	$16d_8^2$
$16d_8$	M_Q	$-M_Q$	$16d_8$
$16d_8$	M_Q	$-M_Q$	$16d_8$

⇈ roton 1		↓ antiroton 2	
$D_1^0(16,e)$	skel.	skel.	$D_2^0(16,e)$
1	$2M_Q$	$-2M_Q$	1
w_{32}^2	$2M_Q$	$-2M_Q$	1
1	$2M_Q$	$-2M_Q$	1
u_{64}	$2M_Q$	$-2M_Q$	1
4	$2M_Q$	$-2M_Q$	4
$8d_{16}^2$	$2M_Q$	$-2M_Q$	$8d_{16}^2$
$8d_{16}$	$2M_Q$	$-2M_Q$	$8d_{16}$
$8d_{16}$	$2M_Q$	$-2M_Q$	$8d_{16}$

⇈ roton 1		↓ antiroton 2	
$D_1^0(32,e)$	skel.	skel.	$D_2^0(32,e)$
1	$4M_Q$	$-4M_Q$	1
w_{64}^2	$4M_Q$	$-4M_Q$	1
1	$4M_Q$	$-4M_Q$	1
u_{128}	$4M_Q$	$-4M_Q$	1
4	$4M_Q$	$-4M_Q$	4
$8d_{32}^2$	$4M_Q$	$-4M_Q$	$8d_{32}^2$

Figure 7: Structural diagrams of three electron neutrinos. The number N_Q of mass quanta per roton equals 64 in all cases. This number is realized having a ring mass of $\pm 2M_Q$ in 32 rings (middle panel, corresponding to the electron model), $\pm 4M_Q$ in 16 rings (right panel), or $\pm M_Q$ in 64 rings (left panel). The neutrino oscillates between these states, because the skeleton energy is exact zero and the total energy is nearly zero in all cases. The weak interaction capability of the electron neutrino in the middle changes to the tauon-like (right) or to the muon-like flavor (left panel).

$$w_{32}^2 + u_{64} - 2 + 0.43(a_{16} - d_{16}) = -4.90 \times 10^{-7}$$
$$w_{32}^2 + u_{64} - 2 + 0.44(a_{16} - d_{16}) = 2.32 \times 10^{-8}$$
$$w_{32}^2 + u_{64} - 2 + 0.45(a_{16} - d_{16}) = 5.37 \times 10^{-7}$$

Dress differences around 10^{-8} would correspond to neutrino masses of some 0.1 eV or below. The neutrino mass does not appear as a constant quantity, it changes also during oscillation.

2.3.3 Tauon and tau neutrinos

The leptons of generation number n = 3 are the tauon, tauon neutrinos and their anti-particles.

The number of fluctuating charges is $N_{fluct} = 104$, each roton carries 52 fluctuating charges. That are exclusively two-cycle charges (energy factor d_{32}). The dress components of the tauon model are

$$D_1(32) = w_{32} + 2w_{64} + u_{128} + 8d_{32}^2 + 36d_{32} + 16 = 68.278631$$
$$\Delta D_1(32) = w_{32} - 1 = 8.029885 \times 10^{-2}$$
$$D_2(32) = 8d_{32}^2 + 36d_{32} + 20 = 68.203485$$

These components provide a representation of the mass of the tauon:

$$m_{\tau,\mathrm{mod}} = \left[128 + 2w_{32} + 2w_{64} + u_{128} - 5 + 72(d_{32} - 1) + 16(d_{32}^2 - 1)\right] * 4m_Q$$
$= 1776.867$ MeV, about $1.4 * 10^{-5}$ smaller than the measured value.

A series expansiom yields the approximation formula

$$m_{\tau,\mathrm{mod}} \approx (128 + 106(32z) + 114(32z)^2 + 142(32z)^3) * 4m_Q = 1776.929 \text{ MeV}$$

The anomalous magnetic moment becomes

$$a_{\tau,\mathrm{mod}} = \Delta D_1(32)/D_1(32)$$

Tauon					
$\upuparrows$ roton 1			$\downarrow$ roton 2		
charges	dress	skel.	skel.	dress	charges
$\ominus$	w_{32}	$4M_Q$	$4M_Q$	1	$\square$
$\oplus\ \oplus$	w_{64}	$4M_Q$	$4M_Q$	1	$\square$
$\ominus\ \ominus$	w_{64}	$4M_Q$	$4M_Q$	1	$\square$
$\oplus\ \ominus\ \oplus\ \ominus$	u_{128}	$4M_Q$	$4M_Q$	1	$\square$
$2\times\oplus\wedge\ominus$	$8+4d_{32}$	$4M_Q$	$4M_Q$	$8+4d_{32}$	$2\times\oplus\wedge\ominus$
$8\times\oplus\wedge\ominus$	$16d_{32}$	$4M_Q$	$4M_Q$	$16d_{32}$	$8\times\oplus\wedge\ominus$
$8\times\oplus\wedge\ominus$	$16d_{32}$	$4M_Q$	$4M_Q$	$16d_{32}$	$8\times\oplus\wedge\ominus$
$8\times\oplus\wedge\ominus$	$8+8d_{32}^2$	$4M_Q$	$4M_Q$	$8+8d_{32}^2$	$8\times\oplus\wedge\ominus$
64 rings per roton, ring mass $4M_Q$					

Figure 8: The structural diagram of the tauon. One of the four states of a Dirac - particle is shown: Positive energies, spin up.Four mass quanta per ring and 64 rings per roton are the main characteristics of the tauon model..

$$= (w_{32}-1)/(w_{32}+2w_{64}+u_{128}+8d_{32}^2\ +36b_{32}+16)$$
$$= 1.179\,48\times 10^{-3},$$

This value is in reasonable agreement with the SM prediction of $a_{\tau,SM} = 1.177\,21 * 10^{-3}$.

An experimental value of a_τ is not available.

There are three variants of the tauon neutrino, see Fig. 9. Each of the three neutrinos has 256 mass quanta per roton. The neutrino variant corresponding to the em-charged tauon is shown in the right panel, it has 64 rings per roton or antiroton with a ring mass of $\pm\ 4M_Q$. The ring masses of the other neutrinos vary by factors of two. This changes the circulation radii, and the corresponding weak charges imitate the weak coupling of an electron (middle panel) or a muon (left panel).

The skeleton masses of roton and antiroton have the same magnitude, both skeleton masses add exactly to zero.The resulting tiny neutrino mass depends on the symmetrization between the dresses of roton and antiroton and changes during oscillation.

2.3.4 Neutrino overview

The model proposed in this article provides nine neutrino models, three per particle generation. The model parameters of the neutrinos are summarized in a table in Fig. 10. The neutrino oscillation occurs along the rows with a certain background color. The change of the weak coupling capability during oscillation

⇈ roton 1		↓ antiroton 2	
$D_1^0(8,\tau)$	skel.	skel.	$D_2^0(8,\tau)$
1	M_Q	$-M_Q$	1
w_{16}	M_Q	$-M_Q$	1
w_{16}	M_Q	$-M_Q$	1
u_{32}	M_Q	$-M_Q$	1
60	M_Q	$-M_Q$	60
$64d_8$	M_Q	$-M_Q$	$64d_8$
$64d_8$	M_Q	$-M_Q$	$64d_8$
$32d_8 + 32d_8^2$	M_Q	$-M_Q$	$32d_8 + 32d_8^2$

⇈ roton 1		↓ antiroton 2	
$D_1^0(16,\tau)$	skel.	skel.	$D_2^0(16,\tau)$
1	$2M_Q$	$-2M_Q$	1
w_{32}	$2M_Q$	$-2M_Q$	1
w_{32}	$2M_Q$	$-2M_Q$	1
u_{64}	$2M_Q$	$-2M_Q$	1
28	$2M_Q$	$-2M_Q$	28
$32d_{16}$	$2M_Q$	$-2M_Q$	$32d_{16}$
$32d_{16}$	$2M_Q$	$-2M_Q$	$32d_{16}$
$16d_{16} + 16d_{16}^2$	$2M_Q$	$-2M_Q$	$16d_{16} + 16d_{16}^2$

⇈ roton 1		↓ antiroton 2	
$D_1^0(32,\tau)$	skel.	skel.	$D_2^0(32,\tau)$
1	$4M_Q$	$-4M_Q$	1
w_{64}	$4M_Q$	$-4M_Q$	1
w_{64}	$4M_Q$	$-4M_Q$	1
u_{128}	$4M_Q$	$-4M_Q$	1
12	$4M_Q$	$-4M_Q$	12
$16d_{32}$	$4M_Q$	$-4M_Q$	$16d_{32}$
$16d_{32}$	$4M_Q$	$-4M_Q$	$16d_{32}$
$8d_{32} + 8d_{32}^2$	$4M_Q$	$-4M_Q$	$8d_{32} + 8d_{32}^2$

Figure 9: Structural diagrams of three tauon neutrinos. The number of mass quanta per roton equals 256 in all cases. This number is realized having a ring mass of $\pm 4M_Q$ in 64 rings (right panel, corresponding to the tauon model), $\pm 2M_Q$ in 128 rings (middle panel), or $\pm M_Q$ in 256 rings (left panel). The neutrino oscillates between these states, because the total energy is nearly zero in all cases. The weak interaction capability changes to the electron-like (middle) or to the muon-like flavor (left panel).

does not change the number $N_Q = 2^{2n+2}$ of mass quanta per roton, n represents the generation number. If one could define the lepton number by using N_Q, the result would be the conservation, not the violation of lepton number by the neutrino oscillation. However, N_Q is currently not observable.

There are 4 leptons per generation, one charged with an em charge and three "uncharged" (in sum neutral) ones, carrying no single em charge, however weak and fluctuating charges. Each lepton model has four Dirac states, this includes particle- / anti-particle-states and spin up / spin down states (see Fig. 19 in [17]). Thus the basic space model provides in three generations 12 leptons with a total of 48 leptonic states. The three em-charged leptons have definte nonzero mass values, where the masses of nine neutrinos are nearly zero. The skeleton masses of neutrinos and of the electron are exactly zero, the total masses of neutrinos are diminished by symmetrization between the dresses of roton and antiroton. Moreover, the neutrino masses change during the oscillation process.

Lepton models have six generations in basic space, corresponding to the six dimensions of 'little string theories', a nongravitational variant of string theories [13]. It is designated as 'nongravitational', because no massless spin two particle (graviton) appears in the theory.

One can speculate, that a tendency to minimize gravity could be the origin of the internal mass compensation in neutrinos.

2.3.5 The fourth lepton generation

A lepton of generation number n = 4 and ring parameter k = 64 would have two rotons with $2k = 2^{n+3} = 128$ rings each. The ring mass would be $\frac{k}{8} m_Q = 2^{n-1} * m_Q = 8m_Q$, so the "charged skeleton" mass of the fourth generation lepton would amount to

$$m_{lept}^S(64) = (w_{64} - 1) * 8M_Q + (128 \pm 128) * 8m_Q \qquad (14)$$

Neutrino oscillation

Roton mass	Parameter	Weak coupling (flavor)		
Number of M_Q	n particle (mass) generation	μ (*weak*)	e (*weak*)	τ (*weak*)
$N_Q = 2^{2n+2}$	n_w weak generation number	$n_w = 1$	$n_w = 2$	$n_w = 3$
$\Downarrow$	ring parameter $k = 2^{n_w+2}$	8	16	32
$\Downarrow$	weak factors $2w_{2k}$ or w_{2k}^2	$2w_{16}$	$2w_{32}$	$2w_{64}$
$\Downarrow$	ring mass $M_r = 2^{n_w-1}M_Q$	$\pm M_Q$	$\pm 2M_Q$	$\pm 4M_Q$
μ , n=1, N_Q=16	ring number N_r =	**16**	8	4
e , n=2, N_Q=64	ring number N_r =	64	**32**	16
τ , n=3, N_Q=256	ring number N_r =	256	128	**64**

Figure 10: Parameters of oscillating neutrino models. The oscillation occurs within rows with the same background colors. The bold numbers mark the fields without oscillation. The weak coupling allows an experimental determination of the neutrino type, characterized by rhe weak generation number n_w.This number changes during oscillation, interpreted as lepton number violation. In case the mass generation number n and the mass eigenstate (number of mass quanta per roton) could be used to define the lepton number, there was no violation.

One gets for a fourth lepton of the muon or tauon type with (128+128) $\Rightarrow$ 256 and $M_Q = 2m_Q$the skeleton mass

$m^S_{lept}(64) = 2(w_{64} - 1) * 8m_Q + 256 * 8m_Q = 6670.9$ MeV

and for a lepton of the electron type with $(128 - 128) \Rightarrow 0$ one would expect around the self-energy of a circulating charge

$m_{lept}(64) \approx 2(w_{64} - 1) * 8m_Q = 9.088$ MeV.

No such lepton could be found experimentally. A lepton of a fourth generation is theoretically excluded within the Standard Model.

The basic space model presented here provides a conjecture concerning the reason, why no lepton with n = 4 exists. The formula for the charged skeleton does not give a hint in this direction. But if we consider the weak interaction, which was neglected up to now, we obtain $a_{128} = 1.412$ for one double ring with two weak charges of equal signs and $a^2_{128} \gtrapprox 2$ for the quadratic form of the energy factors of all four weak charges. This would double the ring mass, which carries the twofold occupied rings. This could possibly result in the production of additional ring masses by the self-energy of circulating charges and therefore cause the collapse of the particle. This would explain that the existence of a lepton of the fourth generation is improbable. Leptons don't play a role for the question, whether an upper limit of particle mass exists or not. A similar situation can be found for baryon models. The only way to answer the question of an upper limit of particle mass is the investigation of the mass scale of mesons.

3 The mass scale of mesons

3.1 The general structure of a meson mass

3.1.1 Parton and anti-parton

The biroton, introduced with lepton models [**?**], works also as the main structural component of mesons and other hadrons. A biroton consists of two rotons with angular momenta (spins) of the magnitudes $\hbar$ and $\hbar/2$, the resulting spin is $\pm\hbar/2$ in leptonic birotons and in the ground states of mesonic birotons. The rotons of a biroton have normally antiparallel spins.

In hadron models, also excited birotons with a total spin of $\pm 3\hbar/2$ may occur. Such excited birotons represent some kind of excited fermion. The biroton has in general the two spin states $s = \frac{1}{2}$ and $s + 1 = \frac{3}{2}$. One of the two rotons of a biroton carries weak or electroweak charges, the other roton is free of such charges.

A meson model consists of a biroton and an anti-biroton, which has negative mass quanta in basic space. In spacetime, all masses are positive and in case of 'mass-symmetric' mesons, the meson mass, observable in spacetime, equals twice the parton mass.

The two spins of a biroton and an anti-biroton can be antiparallel, this results in the spin zero of pseudoscalar mesons. In case of parallel parton spins, the result is the spin one of vector mesons. A pseudoscalar or a vector meson in its ground state consists of four fermions (two rotons and two anti-rotons in the spin state $s = \frac{1}{2}$). This article will be restricted to the masses of pseudoscalar and vector mesons in its ground state. An example of a model of a pseudoscalar meson is shown in Fig. 11.

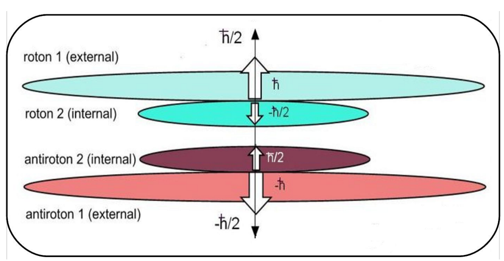

Figure 11: Geometric properties of a meson model consisting of a parton (biroton, green colors) and an anti-parton (anti-biroton, brown colours). The spins of parton and antiparton can be parallel (vector meson) or anti-parallel (pseudoscalar meson, shown in the Fig.). The roton masses are both positive in the parton, and both negative in the antiparton. In spacetime, all masses are positive.

3.1.2 Skeleton and dress of a parton or an anti-parton

The skeleton of a biroton is formed by circulating mass quanta, organized in rings. The skeleton mass represents about 95 % of the total mass of a biroton. The skeleton in mesonic partons is an equivalent not only of the quark content, but also of gluons and their interaction energy, obtained by QCD lattice calculations in the Standard Model. The skeletons of mesons of different generations show the origin of the particle generations in general and of the meson mass scale.

The dress of a biroton consists of circulating charges. The rings of a mesonic parton may carry different kinds of charges, the same as in lepton models (see section 2.2, Fig. 3). The dresses of mesonic and leptonic birotons of the same generation are in principle identically.

The circulating charges can be fixed to a ring mass or they can circulate independently. The independent circulation is a characteristic property of an electromagnetic (em) charge. The other charges circulate together with mass quanta. Electroweak charges are assumed to circulate infintely at the same ring, they do not contribute to the binding between rings by exchange processes.

All charges of a dress circulate at a radius determined by the skeleton. The dress alone does not reveal the origin of the particle generations.

3.1.3 Subcomponents of the skeleton, enhanced mass quanta

Two rotons or anti-rotons belonging to the same biroton have already identical masses and subcomponents. If one roton has N_r rings with the ring mass m_r, so the skeleton mass of one roton is $N_r m_r$ and of the biroton $2N_r m_r$. Some selected mesons are mass symmetric, their skeleton mass equals that of four rotons:

$$m^S_{neson} = 4N_r m_r \quad (mass - symmetric) \tag{15}$$

In lepton models, the ring mass and the ring number have both a 'generation factor' of two per generation. Both quantities are proportional to 2^n, where n means the generation number. Therefore, the number N_Q of mass quanta in models of charged leptons increases quadratic from Muon to Tauon, that means proportional to 2^{2n}.

Such a quadratic increase would not suffice to explain the huge mass increase within the meson mass spectrum. In models of mesons (and generally in hadron models) we assume an additional increase by an enhancement of the mass quanta itself, such as

$m_Q \Rightarrow f_e m_Q$ (in spacetime) and

$M_Q \Rightarrow 2\ f_e m_Q$ (in basic space)

The enhancement factor f_e is determined by the cluster factor w^l_l :

$$f_e \sim w^l_l = \left(\sum_{i=0}^{\infty}(lz)^i\right)^l = \frac{1}{(1-lz)^l}; \quad z = \frac{\alpha}{\pi} \tag{16}$$

One has $f_e = w_l^l$ if each mass quantum of a parton is enhanced, however one has to set $f_e = \frac{1}{2}(w_l^l - 1)$ if only 50% of the mass quanta are enhanced.

The cluster parameter l is an always even, natural number and depends on the generation number n, for instance such as $l = 4 * 2^{n-1}$. The smallest value (at n = 1) is $l = 4$, realized in partons of $\pi-$ mesons[1]). More series of mesons having different characteristic numbers as decribed below in Table 1.

A neutral cluster of l charges ($l/2$ of each sign) produces the cluster factor w_l^l by circulating charges. A cluster is fixed at a certain mass quantum or is jumping between mass quanta as part of an exchange process. The effect on the particle mass is demonstrated in Fig.12 in general form. The quadratic increase is caused by the number of mass quanta $N_Q \sim 2^{2n}$ (dotted red line). The increase by the cluster factor w_l^l is caused by the enhancement of each mass quantum (dotted black curve). The full red line shows the combination of both effects. The skeleton mass of a parton m_p^s increases according to

$$m_p^s \sim 2^{2n} w_l^l m_Q = 2^{2n} m_Q / (1 - l * \frac{\alpha}{\pi})^l; \quad l \sim 2^n \tag{17}$$

The product $2^{2n} w_l^l m_Q$, where n means the generation number, represents the mathematical backbone of the mass scale of meson models. Details are given below, see Table 1.

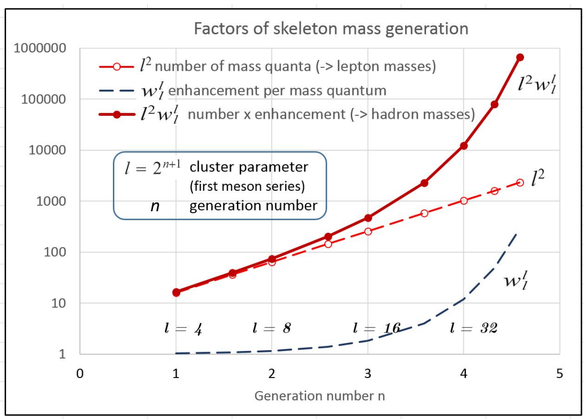

Figure 12: Factors determining the skeleton mass of rotons consisting of enhanced mass quanta $w_l^l m_Q$. The red curve shows the steep incrase of mass with the generation number n and the cluster parameter l. Models of 'main step' mesons have $l = 2^{n+1}$ (= 4, 8, 16. 32) or $l = 3 * 2^n$ (= 6, 12, 24, 48).

The biroton is assumed to be in basic space the minimum geometric realization of a particle. The biroton represents a lepton if $w_l^l = 1$; $l = 0$.

In hadrons, one has $w_l^l > 1$; $l > 0$, and a mesonic biroton represents a parton, the equivalent for the combination of quark and gluon in the Standard

[1]The term 'parton' was first used by Feynman. In this article, 'parton' is used for a combination of quark-equivalents and gluons in models of hadrons.

Model. Two partons, a biroton and an ant-biroton, combine to the model of a meson.

The exchange of charge clusters realizes the binding between parton and anti-parton. This mechanism provides the 'glue' between partons / anti-partons, this is the equivalent to the binding of quarks and anti-quarks by the exchange of gluons in the Standard Model.

3.1.4 Charge clusters and the strong interaction

The exchange of charge clusters between mass quanta constitutes the strong binding force between birotons. The capability of a biroton to strong coupling is determined by the the quantity α_{s_eff} of its enhanced ring masses, it is proportional to the reciprocal of the cluster factor (enhance factor) $f_e = w_l^l$ and therefore depends on the parameter l :

$$\alpha_{s\_eff}(l) = \frac{1}{w_l^l} = (1 - lz)^l \ ; \quad z = \frac{\alpha}{\pi} \tag{18}$$

Here $\alpha = \alpha_{em} \approx 1/137$ means the finestructure constant, the constant of the electromagnetic (em) coupling. The product of the strong coupling constant and an enhanced mass quantum results in the mass quantum without enhancement.

The effective strong coupling α_{s_eff} shows characteristics of containment (high values at low energies) and of asymptotic freedom (low values at high energies). This resembles the dependence of $\alpha_s(Q)$ from the collosion energy Q according to QCD-calculations, see Fig. 13. However, the energies on the abscissae of Fig. 13 are different quantities. The red crosses refer to invariant energies of particles 'at rest'. The black linesthe and black points refer to collision energies Q (after [?]). Nevertheless, the general behavior of the curves agree, and the coupling constant, normalized to the mass od the Z particle, is nearly identical for both relationships.

The value of the Standard model, related to the mass of the Z - boson, is in agreement with QCD calculations and amounts to

$$\alpha_s(M_Z) = 0.1189 \pm 0.0010 \tag{19}$$

The value proposed in this article for the basic-space model of the Z - boson amounts to

$$\alpha_s(48/2) = 2/w_{48}^{24} = 0.11718 \tag{20}$$

This model value is obtained using the assumption of cluster splitting (see subsection 3.3.3 and Fig. 21 below).

Fig. 14 shows an overview of numerical values of the strong alpha coupling according to the basic space models (blue curve). The dashed black line marks the values $\alpha_s(48/2)$ as well as $\alpha_s(M_Z)$.

3.1.5 Contributions to the dress

The dresses of mesonic partons contain the same types of circulating charges as the dresses of lepton models (see Fig. 3):

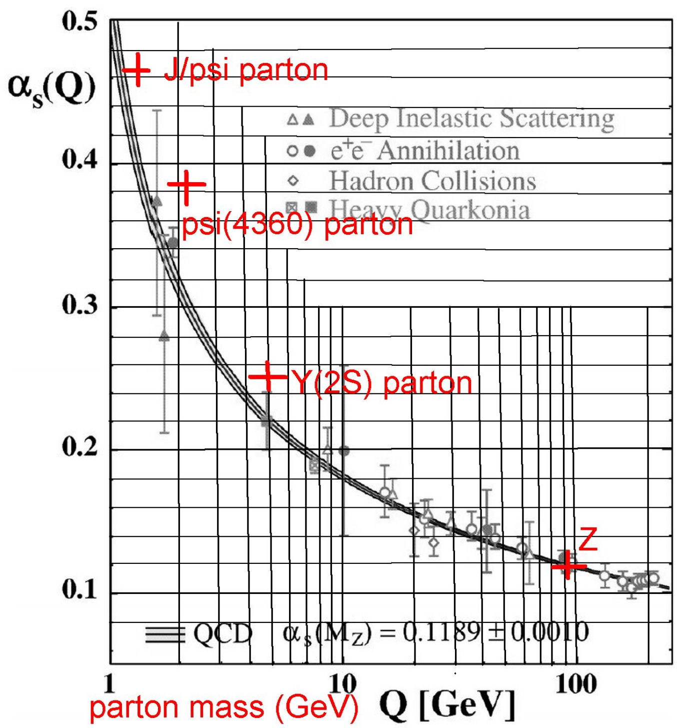

Figure 13: Comparison of the strong alpha values obtained in QCD, confirmed by different experimental methods, and selected model results. The abscissae are not identical: The collision energy Q is measured externally. in contrast the parton mass is an intrinsic property of a particle.

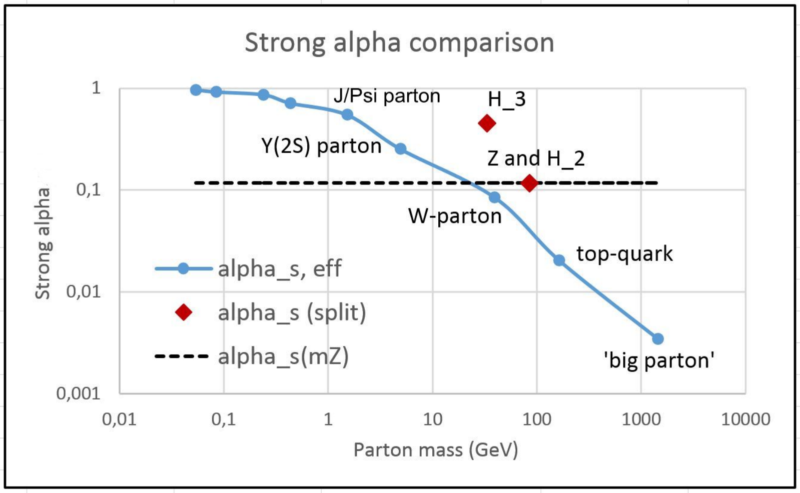

Figure 14: The strong alpha of models of 'main step' mesons are shown in dependence on the parton masses. The big parton, not realized in nature, would have an extremely small strong alpha, smaller than the electromagnetic coupling. The assumed cluster splitting at $l = 48$ leads to a strong coupling of Z^0 in agreement with the value of the Standard Model $\alpha_S(M_Z)$.

- An electromagnetic (em) charge contributes $(2w_k - 2) * f_n f_e m_Q$ to the parton mass (a single em charge contributes its self-energy from the circulation in the eigenspace, this energy is preserved and appears with a factor 2 in spacetime [17])

- Weak charges contribute $(2w_{2k} + u_{4k} - 3) * f_n f_e m_Q$ or $(w_{2k}^2 + u_{4k} - 2) * f_n f_e m_Q$ to the mass of each parton. The weak group comprises two doubly charged rings $(+\,+)$ and $(-\,-)$, and a compensation ring containing the same number of positive and negative charges in alternating succession. The weak group is attached to one of the both rotons, it does not influence the electromagnetic properties of the particle.

- Fluctuating charges change the ring after one, two or three cycles, and contribute $(2a_k - 2) * f_n f_e m_Q$ or $(a_k^2 - 1) * f_n f_e m_Q$, where the energy factors a_k could be replaced by b_k or d_k. The **number of fluctuating charges** N_{fluct} per parton and per roton has to be even, thus fluctuating charges do not influence the electromagnetic properties of the particle. The number N_{fluct} of exact mass-symmetric mesons is a multiple of eight, each of the four rotons carries the same even number of charges.

The circulating charge clusters are considered to belong to the skeleton.

3.1.6 Series of skeleton masses of meson models

The skeleton masses of mesons do not increase along a single mass scale, in contrast we find four different series going up.

The **ring number per parton** $2N_r$ increases with the generation number n according to two different rules:

$2N_{r4}(n) = 4 * 2^n = 8$, 16, 32, and 64 rings per parton (n = 1. ...4) and
$2N_{r6}(n) = 6 * 2^n = 12$, 24, 48, and 96 rings per parton.

The **ring mass** m_r contains the universal mass quantum m_Q, derived from empirical muon data and lepton model regularities. The ring mass depends on the generation number n and as well on the cluster parameter l :

$$m_r(n,l) = f_n f_e m_Q = 2^{n-1} w_l^l m_Q \tag{21}$$

$$m_{red}(n,l) = f_n f_e m_Q = 2^{n-1} * \frac{1}{2}(w_l^l + 1) m_Q \tag{22}$$

$$f_n = 2^{n-1} \tag{23}$$

The **generation factor** takes the values $f_n = 2^{n-1} = 1$, 2, 4, and 8 for n = 1...4;

the **enhancement factor** f_e describes the energy increase of a ring mass caused by circulating charge clusters:

$$f_e = w_l^l \ ; \qquad f_e = \frac{1}{2}(w_l^l + 1) \tag{24}$$

The **mass quantum number** $2N_Q$ of mass quanta per parton or N_Q per roton is now given by the expressions

$N_Q = 2 * 2^{2n} = 8$, 32, 128, and 512 for n = 1...4 or
$N_Q = 3 * 2^{2n} = 12$, 48, 192 and 768

The masses of four series of ground state mesons are constructed using different formulas for the ring number N_r and the ring mass m_r, see Table 1.

Table 1: Mass formulas for mesonic parton masses in skeleton approximation

Series →	1	2	3	4
$2N_r(n)$	$4 * 2^n$		$6 * 2^n$	
$m_r(n,l)$	$2^{n-1} w_l^l m_Q$; reduced:		$2^{n-1}(\frac{1}{2} w_l^l + \frac{1}{2}) m_Q$	
$2N_r * m_r$	$4 * 2^{2n} w_l^l m_Q$		$6 * 2^{2n} w_l^l m_Q$	
N_Q	$4 * 2^{2n}$		$6 * 2^{2n}$	
l	$4 * 2^{n-1}$	$5 * 2^{n-1}$	$6 * 2^{n-1}$	$7 * 2^{n-1}$
$l(n) \leqslant 40$	4, 8, 16, 32	10, 20, 40	6, 12, 24	14, 28

The series 2 and 4 have no members in the first generation, because the cluster parameter l has to be always even. Series 4 doesn't have a meson with $l = 56$ because of cluster splitting (see the next sections). In the last row the cluster parameter l is restricted to $l(n) \leqslant 40$, although the four definitions of l do not contain any restriction for the generation number n. The restrictions to

$n \leqslant 4$ and $l(n) \leqslant 40$ come from empirical facts and will be discussed in section 3.2.3.

Several skeleton masses of partons belonging to the series 1 and 3 described in Table 1 form mass symmetric meson models, called 'main step' mesons. They correspond in part to the masses of 'quarkonia' in the Standard Model of particles. The dual space model provides the meson masses of the first series in skeleton approximation according to

$$2m_P^S(\text{series}\,1) = 4N_r * m_r = 8 * 2^{2n} w_l^l m_Q \quad (25)$$
$$l = 4 * 2^{n-1}; \quad n = 1...4 \quad (26)$$

This formula yields the skeleton masses of K- mesons, the J/Ψ meson, and also the $W^{\pm}-$ particle.

The mass-symmetric mesons of series 3 have the skeleton masses

$$2m_P^S(\text{series}\,3) = 4N_r * m_r = 12 * 2^{2n} w_l^l m_Q \quad (27)$$
$$l = 6 * 2^{n-1}; \quad n = 1...3 \quad (28)$$

This formula yields the skeleton masses of K* - mesons at n = 2, and of the $\Upsilon(2S)$ - particle at n = 3.

Numerical examples of the skeleton and the full masses of such 'main step' meson models will be derived in the following sections.

3.2 Meson models in four generations

3.2.1 Model masses of π mesons, the first generation

Models of π^0 and $\pi^{\pm}$ - mesons have a different number of clusters per mass quantum, see Table 2 (appendix). We listed the enhancement factor f_e and its reciprocal, the strong coupling constant $\alpha_s = 1/f_e$ for partons of the first generation (n = 1).

The pion masses are the sums of two different parton masses, they are always mass - asymmetric.

The structural steps from the mass quantum to the meson mass of the $\pi^{\pm}$ is shown in Table 3 (appendix). The pion - models have the smallest ring masses of all meson models.

The parton masses in generation n = 1 are given in Table 4 (see appendix).

The components of a parton mass are the skeleton and the dress contributions. Both contributions are summarized in the full parton models in Table 4. The main parts come from the skeletons, represented by the factors 16 (l = 4) or 24 (l = 6) in the mass formulas.

In Table 5 all dress contributions are separated, belonging to parton masses of the $\pi^{\pm}-$ meson. The mass differences for one-cycle fluctuating charges (energy - factor b_k) and for three-cycle charges (factor a_k) are shown.

Table 5: Dress contributions to the masses of $\pi_4^{\pm}$ and $\pi_6^{\pm}$ partons

l	Contrib.	mass formula	MeV
4	em	$(2w_4 - 2) * w_4^4 m_Q$	$6.333\,435 \times 10^{-2}$
4	$weak$	$(2w_8 + u_{16} - 3) * w_4^4 m_Q$	$-2.108\,275 \times 10^{-3}$
4	$fluct(b)$	$2(b_4 - 1) * w_4^4 m_Q$	$6.274\,59 \times 10^{-2}$
4	$fluct(a)$	$2(a_4 - 1) * w_4^4 m_Q$	$6.333\,43 \times 10^{-2}$
6	em	$(2w_6 - 2) * w_6^6 m_Q$	0.100 029
6	$weak$	$(2w_{12} + u_{24} - 3) * w_6^6 m_Q$	$-4.695\,77 \times 10^{-3}$
6	$fluct(b)$	$2(b_6 - 1) * w_6^6 m_Q$	0.098 635
6	$fluct(a)$	$2(a_6 - 1) * w_6^6 m_Q$	0.100 029

A graphical representation of the parton masses of the π mesons in dependence on the cluster parameter is given in Fig. 15.

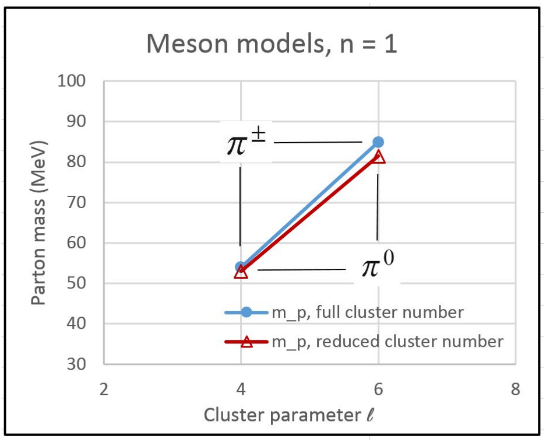

Figure 15: Models of π - mesons. The first particle generation n = 1 contains only the π - mesons (mass-asymmetric). The ring masses equal to one cluster per mass quantum $w_l^l m_Q$ in the model of the $\pi^{\pm}$ or to one cluster per two mass quanta $(\frac{1}{2} + \frac{1}{2} w_l^l) m_Q$ in the model of the π^0 pion. The first particle generation has the cluster parameters $l = 4$ and $l = 6$.

3.2.2 Model masses of mesons of the second and third generation

In the second generation, there exist besides the main step mesons with l = 8 and l = 12 also mesons with l = 10 and l = 14, see Fig. 16. This results in three mass-symmetric and several mass-asymmetric mesons, that cannot be discussed in full.

The main steps of particle structure are represented by the K^0 at $l = 8$ and by the K^{*0} - mesons at $l = 12$. The structural composition of these two mesons is given in Table 6 (see appendix). Parton and antiparton of the K^0 and K^{*0} - mesons are identically with respect to all mass contributions.

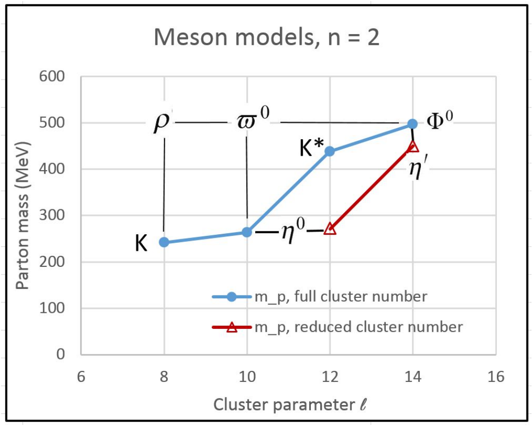

Figure 16: Models of mesons of the second generation. K-, K*-, and Φ^0 - mesons have mass-symmetric models. The models of η^0 and η' have one parton with a reduced cluster number (one cluster per two mass quanta). The cluster parameters of the second generation are $l = 8$ and $l = 12$ for 'main step' mesons. In general, l is an even number, thus $l = 10$ and $l = 14$ are possible.

The $K^{\pm}$ - and $K^{*\pm}-$ mesons are symmetric with respect to the skeleton masses of parton and antiparton. However, the mass contribution of the em charge causes a slight asymmetry because the em charge is attached only to one of the partons (either $\ominus$ to the parton or $\oplus$ to the antiparton). Therefore we give the mass contributions in the Tables 7 and 8 for parton and antiparton separately (see appendix).

A large number of meson models can be constructed in the third generation, see the graphical representation in the Figs. 17 and 18.

The mass contribtions to models of the 'main step' mesons at $l = 16$ and $l = 24$ are contained in Table 9 (see appendix).

The ground states of different Ψ, Υ and η - mesons can be constructed in a similar way, using cluster parameters l between 16 and 26.

The partons of the second and third generation may combine, the result are different D- and D* as well as B- and B* - mesons. Some members of these families are shown in Fig. 19 as combinations of two partons, one coming from the generation n = 2 and one from n = 3.

Some mesons represent 'tetraquark candidates', they can be interpreted as consisiting of four quarks, because the quark - antiquark picture for normal mesons does not fit [24].[23]. The dual space model proposed in this article can be adapted by combining partons with appropriate skeleton masses, examples of models of tetraquark candidates are contained in Table 10 in the appendix.

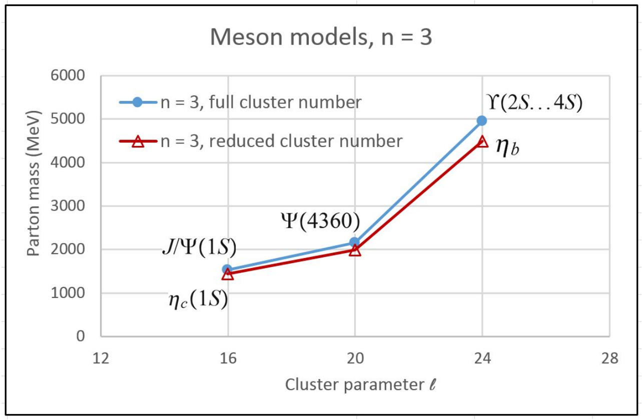

Figure 17: Models of selected third generation mesons. The charmed and bottomed η - mesons consist of partons with reduced cluster numbers (one cluster per two mass quanta). The cluster parameters of the third generation are $l = 16$ and $l = 24$.for 'main step' mesons.

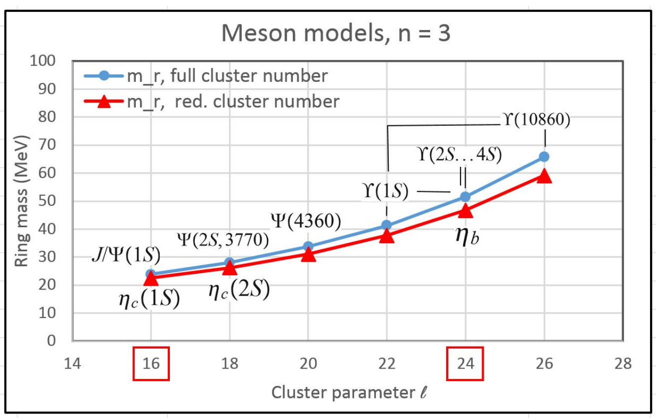

Figure 18: Ring masses of selected third generation mesons. Different masses of the models can be caused by different cluster parameters (examples; $\Psi(1S)$, $\Psi(2S)$ ...) or by different numbers of fluctuating charge pairs (examples: $\Upsilon(2S)$ to $\Upsilon(4S)$). 'Main step' mesons have the cluster numbers $l = 16$ or $l = 24$.

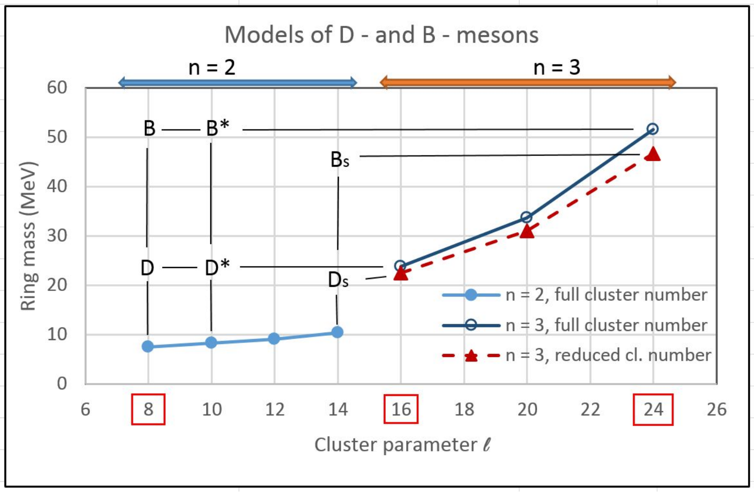

Figure 19: Models of D- and B- mesons as examples of mass - asymmetric mesons consisting of a n = 2 as well as a n = 3 parton. Cluster parameters are even numbers between $l = 8$ and $l = 24$.

3.2.3 The fourth generation - mesons and elementary bosons

The fourth generation appears not fully populated in comparison to the generations two and three. Moreover, in the middle of the fourth generation the model construction known from the other generations seems to fail. Only a radical change, the assumption of a cluster splitting, can remedy the situation. This is the reason to presume an upper limit, a realization threshold for particle masses. In the Fig. 20 and the following text the details of this presumption will be explained.

The $W^{\pm}$ - particle The model of the $W^{\pm}$ appears as that of a normal vector meson consisting of two spin-parallel partons, one of them carrying an integer em charge. They have 28 fluctuating charges (14 charge pairs) per parton, which don't contribute to the overall charge of the parton. The partial spins are
2 x $\frac{1}{2}\hbar = \hbar$ similar to the $K^{*\pm}$ and other vector mesons. The partial masses amount to

$$(128 + 2w_{64} + u_{128} - 3 + 14(a_{32}^2 - 1)) * 8w_{32}^{32}m_Q + (2w_{32} - 2) * 8w_{32}^{32}m_Q$$
$$= 40211.16 \text{ MeV}, \quad N_{fluct} = 28$$
$$(128 + 2w_{64} + u_{128} - 3 + 14(a_{32}^2 - 1)) * 8w_{32}^{32}m_Q$$
$$= 40161.67 \text{ MeV}, \ N_{fluct} = 28$$

The sum becomes
$m_{\text{mod}}(W^{\pm}) = 40161.67 + 40211.16 = 80372.83$ MeV,$N_{fluct} = 56$
This result agrees with the observed value within the measuring uncertainty:

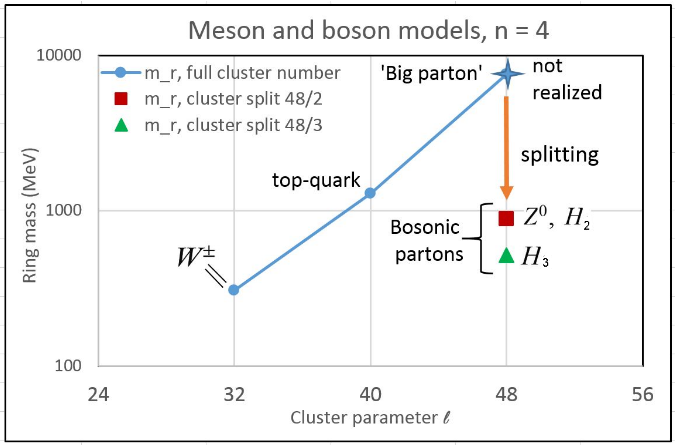

Figure 20: Models of fourth generation particles. The partons of the $W^{\pm}-$particle ($l = 32$) and the top-quark ($l = 40$) can be constructed using regular formulas..At $l = 48$, however, the mass value predicted by the model (the 'big parton') cannot be detected experimentally. The cluster with 48 charges was splitted into two pieces (24 charges) or into three pieces (16 charges per cluster). Only with the assunption of splitting can be constructed models of the Z^0 and the H^0 (Higgs.boson), where regular formulas (without splitting) give no result.

$m_{\rm exp}(W^{\pm}) = 80.379 \pm 0.012 \;\; GeV$
The skeleton mass contains $2 * 128 = 256$ rings with the ring mass
$8w_{32}^{32}m_Q = 308.1478$ MeV, in total
$m_{\rm mod}^{S}(W^{\pm}) = 2 * 128 * 8w_{32}^{32}m_Q = 78885.846$ MeV.
The skeleton provides around 98 % of the particle mass.

The top quark The model of the top-quark appears as a fermionic parton (a biroton) existing in two variants, with or without an integer em-charge:

$$m_{\rm mod}(t^0) = (128 + 2w_{80} + u_{160} - 3 + 14(a_{40}^2 - 1) + 32(a_{40} - 1)) * 8w_{40}^{40}m_Q = 172765\ MeV\ ; \quad N_{fluct} = 60$$
$$m_{\rm mod}(t^+) = (128+2w_{40}+2w_{80}+u_{160}-5+14(a_{40}^2-1)+30(a_{40}-1))*8w_{40}^{40}m_Q = 172765\ MeV\ ; \quad N_{fluct} = 58$$

Both model values of the $t-$mass agree within the measuring uncertainty with the measured value:
$m_{\rm exp}(t) = 172.76 \pm 0.3\ GeV$
The broken charge of $+\frac{2}{3}e$ according to the quark-model can be obtained using the assumption, that the two variants exist always with the relation $2:1$.

The top quark has the maximum parton mass realized in nature. We remark, that the numbers N_{fluct} of fluctuating charges in top quark models are not unique, other models with different (always even) numbers N_f are possible within the model.

The skeleton mass contains 128 rings with the ring mass $8w_{40}^{40}m_Q = 1286.\ 45$ MeV:

$m_{\rm mod}^S(t) = 128 * 8w_{40}^{40}m_Q = 164665.\ 2$ MeV, the skeleton mass amounts to more than 95 % of the full mass.

The Z^0 particle, two fractions of splitted charge clusters The attempt to construct the model of the Z^0 similar to the model of the $W^\pm$ does not succeed. The ring mass $8w_{32}^{32}m_Q$ would require an unusual high number of fluctuating charges to reach the experimental Z^0 mass. A change from $l = 32$ to $l = 34$ would result in a skeleton mass higher than the experimental Z^0 mass. Therefore one needs a very special model construction, unique to the Z^0 particle.

Accordimg to this unique construction, the model of the Z^0 appears as a bosonic parton with spin $\hbar$. This seems to be justified by assuming a special ring mass of $8(2w_{48}^{24}m_Q) = 888.\ 28$ MeV. This ring mass is the result of a cluster splitting of the calculated very high ring mass of the 'big parton' of $8w_{48}^{48}m_Q = 7580.\ 3$ MeV. The splitting of a cluster with $l = 48$ charges, attached to a mass quantum, is assumed to result in two mass quanta enhanced by one half of the clustered charges:

$$w_{48}^{48}m_Q \rightarrow 2w_{48}^{24}m_Q \qquad (29)$$

The skeleton mass contains 96 rings and amounts to

$m_{\rm mod}^S(Z^0) = 96 * 8(2w_{48}^{24}m_Q) = 85275.\ 04$ MeV. This is one half of the ring number of the big parton (P_{big}). It would have the skeleton mass

$m_{\rm mod}^S(P_{big}) = 2 * 96 * 8w_{48}^{48}m_Q = 1455421\ MeV = 1.455\ TeV$. The number of mass quanta of the model of the Z^0 ($N_Q = 1536$) is identically with that of the big parton.

The full model mass of the Z^0 particle equals

$$m_{\rm mod}(Z^0) = (96 + 2a_{96} + u_{192} - 3 + 23(a_{48}^2 - 1) + 4(a_{48} - 1)) * 8(2w_{48}^{24}m_Q)$$
$$= 91187.\ 93\ MeV\ ; \qquad N_{fluct} = 50$$

The mass of the model lies within the neasuring uncertainty of the observed mass of the Z^0 - boson:

$m_{\rm exp}(Z^0) = 91.1876 \pm\ 0.0021\ GeV$.

The Z^0 appears as a single parton (a biroton similar to a lepton), with two bosonic rotons of 48 rings each. The partial spins are $+2\hbar$ and $-\hbar$ and give the total spin $\hbar$. The radii of circulation have twice the values as given for r_1 and r_2 in (3).

The skeleton mass amounts to

$m_{\rm mod}^S(Z^0) = 96 * 8(2w_{48}^{24}m_Q) = 85275$ MeV, about 94 % of the full mass.

According to this model, the Z^0 appears as a bosonic parton with spin $\hbar$.

The Higgs boson H^0, two and three cluster fractions Model masses of the Z^0 and the Higgs bosons can be constructed assuming a splitting of clusters containing 48 charges into clusters containing only 24 or 16 charges, see Figs.

21, 22, and 23. A model of an observed particle does not exist, which would need $l = 48$ or more charges per cluster. So we presume an upper limit of mass (ring mass, parton and particle mass).

Splitting	Unsplitted
charges per cluster	$l = 48$
enhancement factor	$w_{48}^{48} = 291.3$
strong alpha	$\alpha_s = 1/w_{48}^{48} = 0.00343$
$\alpha_s : \alpha_{em}$ relation	$\alpha_s < \alpha_{em} \approx 0.0073$
'big parton'	not realized

Splitting	2 fractions	3 fractions
charges per cluster	$l = 48 \Rightarrow 2 * 24$	$l = 48 \Rightarrow 3 * 16$
enhancement factor	$2w_{48}^{24} = 34.13$	$3w_{48}^{16} = 19.89$
strong alpha	$\alpha_s = 2/w_{48}^{24} = 0.11718$	$\alpha_s = 3/w_{48}^{16} = 0.45256$
$\alpha_s : \alpha_{em}$ relation	$\alpha_s > \alpha_{em} \approx \frac{1}{137} \approx 7.3 \times 10^{-3}$	
partons	realized in Z^0 ; H_2	realized in H_3

Figure 21: Enhancement factors and strong coupling constants caused by cluster splitting. Cluster splitting leads to an essential decrease of the enhancement factor and an increase of the strong coupling constant. The copling constant of two fractions agrees well with the experimental value of the strong coupling constant at the mass of the Z - boson.

The model of the H^0 appears as a pseudoscalar boson composed of two bosonic partons with antiparallel spins of $\pm\hbar$. One of the partons resembles the Z^0 (cluster split into two partial clusters) and the second parton shows another cluster split into three partial clusters:

$$m_{\text{mod}}(H_2 \ni H^0) = (96 + 2a_{96} + u_{192} - 3 + 48(b_{48} - 1)) * 8(2w_{48}^{24} m_Q) = 90055.35 \text{ MeV}, \; N_{fluct} = 48$$

$$m_{\text{mod}}(H_3 \ni H^0) = (64 + 2a_{96} + u_{192} - 3 + 32(b_{48} - 1)) * 8(3w_{48}^{16} m_Q) = 34982.51 \text{ MeV}, \; N_{fluct} = 32.$$

$$m_{\text{mod}}(H^0) = 90055.35 + 34982.51 = 125037.86 \text{ MeV}, \; N_{fluct} = 80 \text{ for two bosonic partons.}$$

The model mass agrees well with the measured value:

$m_{\text{exp}}(H^0) = 125.10 \pm \; 0.14 \; GeV$

The skeleton masses of both partons add to the total skeleton mass $m_{\text{mod}}^s(H^0) = 96 * 8(2w_{48}^{24} m_Q) + 64 * 8(3w_{48}^{16} m_Q) = 118395.8 \; MeV$; the skeleton mass provides about 95 % of the full particle mass.

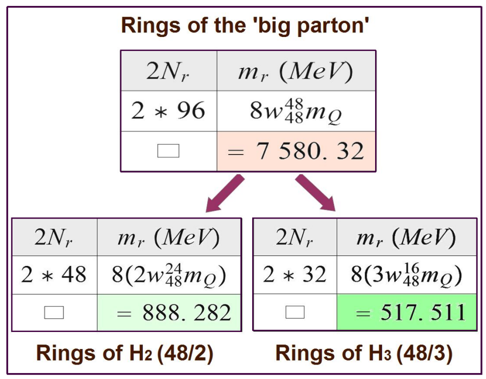

Figure 22: Effects of cluster splitting on ring numbers and ring masses. The ring masses become essentially reduced by the splitting.

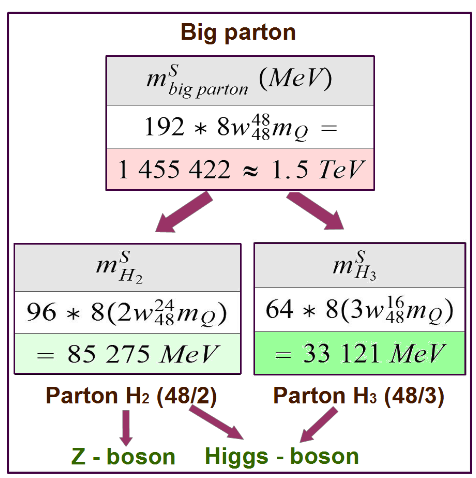

Figure 23: Effects of cluster splitting on parton masses. The big parton (1.5 TeV) does not appear in nature, instead the partons H_2 (85.3 GeV) or H_3 (33.1 GeV) appear, realized as Z^0 or as a parton of the Higgs-boson.

The partial spins $\pm \hbar$ of the two partons are antiparallel, they add to zero. According to this model, the H^0 is a pseudoscalar boson.

3.3 Overview on 'main step' mesons of four generations

The model proposed in this article provides an analytic solution for the skeleton meson masses in different generations. The solution allows the calculation of the masses of different series of partons which yield mass-symmetric as well as mass-asymmetric mesons. The special case of 'main step' mesons includes mass-symmetric mesons of generation n with selected charge parameters l,increasing proportional to 2^n, see Fig. 24.

		Skeleton masses (energy equivalents in MeV)				Exp.
		parton, skeleton formula		parton	meson	meson
n	l	$l = 2^{n+1}$	$l = 3 * 2^n$	MeV	MeV	
1	4	$2^4 * w_4^4 m_Q$		54	139	π
1	6		$3 * 2^3 * w_6^6 m_Q$	85		
2	8	$2^6 * w_8^8 m_Q$		242	484	$K, \overline{K}$
2	12		$3 * 2^5 * w_{12}^{12} m_Q$	438	876	$K^*, \overline{K^*}$
3	16	$2^8 * w_{16}^{16} m_Q$		1526	3052	J/Ψ
3	24		$3 * 2^7 * w_{24}^{24} m_Q$	4949	9898	$\Upsilon(2S)$
4	32	$2^{10} * w_{32}^{32} m_Q$		39443	78 886	$W^{\pm}$
4	40	$2^{10} * w_{40}^{40} m_Q$	$l = 2.5 * 2^n$	164665	329 GeV	$t\,\bar{t}$
4	48		$3 * 2^9 * w_{48}^{48} m_Q$	1.46 TeV	2.91 TeV	
				'big parton', not realized		

Figure 24: Main step partons and mesons of four generations. The row with green background color contains the top quark and represents the highest parton mass realized in nature. The row with red background color contains the 'big parton', the first, lightest 'main step' parton which could not be observed.

A graphical log-log representation of the parton masses of main-step mesons is shown in Fig. 25. The light and heavy quarks are assigned to certain particle generations in the lower panel of Fig. 25.

An overview is given in Fig. 26 on the mass scale of mesons, where mathematically four generations can be identified.

The fourth generation, not defined in the Standard model, is populated by the $W^{\pm}$, Z^0 and H^0 bosons as well as by the top-quark.

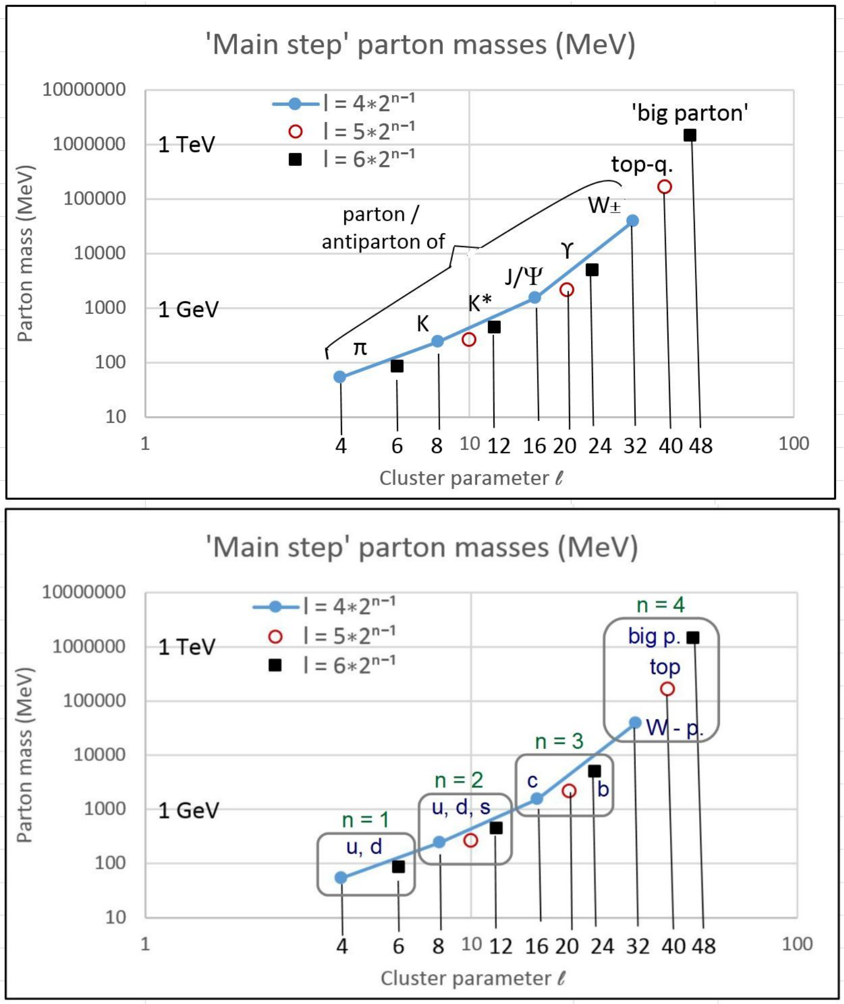

Figure 25: The mass scale of 'main step' mesons in dependence on the cluster parameter l. The skeleton masses of partons of gound state mesons are shown in dependence on l in a double logarithmic representation (upper panel). The same parton masses are equivalents of quark / gluon masses (for heavy quarks) and can be assigned to four generations (generation number n, lower panel).

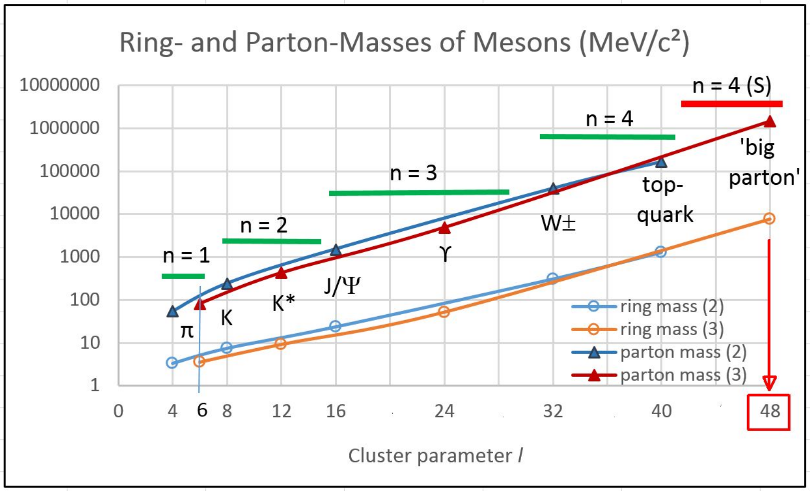

Figure 26: Ring masses and parton masses of 'main step' mesons. The 'main step' mesons of the first series have $l = 2^{n+1}$, marked as mass(2). The third series has $l = 3*2^n$ and is as mass(3) indicated. The top quark has $l = 2^5+2^3 = 40$. The fourth generation is divided into a 'green' region n = 4 without and a 'red' region n = 4(S) with cluster splitting.

4 Conclusions

The geometric particle model based on the biroton structure, already developed for em-charged leptons [17], could be applied to neutrinos and mesons. The mass quantum $m_Q \approx \frac{1}{32} m_\mu = 3.3\ MeV/c^2$, derived from empirical muon data and from model assumptions concerning leptons, turned out to be applicable for mesons too. The mass quantum is used as a universal constant. The family structure of leptons as well as mesons could be described by an analytical function.

At each of three lepton generations, three neutrinos are proposed in addition to one lepton em-charged lepton. The three neutrinos belonging to one mass eigenvalue (a constant number of mass quanta per roton) develop weak coupling capabilities (flavor eigenvalues) during oscillation. These coupling capabilities are caused by changing the radius of circulation and correspond to that of other leptons.

The meson models living in basic space consist of a biroton and an anti-biroton with parallel or antiparallel spins, providing models of vector- or pseudo-scalar mesons. The calculation of mass contributions from the skeleton and the dress of birotons is also analogous to that in lepton models. The dresses carry the same types of circulating charges as in lepton models.

The mass quanta in models of mesons and other hadrons are enhanced by charge clusters, attached to the mass quanta. The charge clusters contain different numbers l of charges, l is the so called 'cluster parameter'. The cluster parameter is always even and the clusters are electrically neutral, they are assumed to be the glue between parton and anti-parton of a meson. The reciprocal of the energy factor of a cluster seem to play the role of an effective strong coupling constant.

An analytical formula is given for the skeleton masses of mesons, starting with a first particle generation containing pion masses and reaching to a fourths generation containing the masses of the $W^{\pm}$ - boson and the top quark.

The masses of D - and B - mesons as well as of different tetraquark - candidates can be modelled using different cluster - and ring - parameters for parton and antiparton (mass-asymmetry).

Two facts reveal serious problems in the model construction:

- The regular formulas describing the masses of particle models cannot be used successfully to construct models of the Z - boson and the Higgs-particle.
- A mesonic biroton with a cluster parameter $l = 48$ would have a calculated skeleton mass of about 1.45 TeV (called the 'big parton'). The big parton was not detected experimentally.

Both problems of the proposed meson model, the missing realization of the predicted 'big parton' and the missing model construction for the Z - boson and the Higgs-particle, can be resolved at once by a single assumption. We assume a splitting of the charge clusters at the level of the 'big parton' into two or three pieces. The splitting occurs in cases where the number l of charges per cluster reaches or exceeds a certain threshold, such that $l \geqq 48$. The splitting hypothesis applied to the predicted model of the 'big parton' results in plausible models of the Z^0 - boson (splitting of each cluster into two pieces) and of the Higgs-boson (splitting into two pieces in one parton and into three pieces in the other parton).

The splitting hypothesis can be interpreted as a prediction of a general upper limit of the formation of particle masses. The mass of the top quark (possibly as part of toponium, $t\,\bar{t}$) appears, according to this hypothesis, as the highest mass of a biroton realized in nature.

The conjecture is suggestive, that the energy region above the toponium threshold, that means above about 400 GeV, is empty of particles, an absolute 'desert'.

The consequences of this conjecture could influence the planning of future experiments in high energy physics.

Experiments directed towards the investigation of particles already discovered have a positive perspective and are not affected by the splitting hypothesis. This would include neutrino programmes and experiments proposed with several colliders, for instance the Linear Collider LCF [27].

Experiments intended to discover new particles in the TeV region appear doubtful. The slogan ' the higher the energy, the better the chance to find new physics' is actually not convincing, although mainstream.

The CERN strategy group expressed the opinion [18]:

"Nature hides the secrets of the fundamental physical laws in the tiniest nooks of space and time. By developing technologies to probe ever-higher energy and thus smaller distance scales, particle physics has made discoveries that have transformed the scientific understanding of the world. Nevertheless, many of the mysteries about the universe, such as the nature of dark matter, and the preponderance of matter over antimatter, are still to be explored."

This statement is used to underpin the necessity to build the 'Future Circular Collider' FCC near the LHC.

"The electron–positron Future Circular Collider (FCC-ee) is recommended as the preferred option for the next flagship collider at CERN". [33]

The argumentation for machines to reach ever higher collision energies is widespread and present also in strategies of the high energy physics in the US [34], and in asia (Japan, China, Korea).

However, the 'tiniest nooks of space and time' could be empty. The secrets of the fundamental physical laws could be hidden in an extra space, not accessible to direct experiments. This would render the FCC a big misinvestment. This represents an important question because of the high costs of new colliders [14].

It is recommended, not to pursue a strategy to reach ever higher energies. However, possibly the prediction of a 'desert' at energies above the top quark threshold could be proved wrong, and then the main point of this article would become obsolete.

5 Declarations

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6 Appendix

The appendix contains Tables and formulas of complete model structures of selected mesons, mainly the ground states forming the 'main step' mesons of different generations.

Table 2: Enhanced ring masses $f_e m_Q$ (MeV)
of the first generation (n = 1), n means the generation number.

l	symbol	f_e formula	f_e value	$\alpha_s = 1/f_e$	$f_e m_Q$ (MeV)
4	$m_P^S(\pi^0_{4red})$	$\frac{1}{2}(w_4^4+1)$	$1.019\,022$	0.981	$3.314\,725$
4	$m_P^S(\pi^\pm_4)$	w_4^4	$1.038\,045$	0.963	$3.376\,602$
6	$m_P^S(\pi^0_{6red})$	$\frac{1}{2}(w_6^6+1)$	$1.043\,928$	0.958	$3.395\,741$
6	$m_P^S(\pi^\pm_6)$	w_6^6	$1.087\,857$	0.919	$3.538\,634$

Table 3: Structural components of the model of a $\pi^\pm$ - meson
(energy equivalents of masses in MeV)

Quantity	$l=4$; $N_r=16$	$l=6$; $N_r=24$
Naked mass quantum m_Q	$3.252\,848$	
Enhanced mass quantum $w_l^l m_Q$	$3.376\,602$	$3.538\,634$
Ring mass $2^{n-1} w_l^l m_Q$; $n=1$	$\Uparrow$	$\Uparrow$
Parton skeleton mass $N_r w_l^l m_Q$	$54.025\,632$	$84.927\,216$
Parton full mass without em charge	$54.150\,19$	$85.422\,66$
Parton full mass with em charge	$54.150\,19$	$85.422\,66$
Meson mass (model)	$139.572\,85$	
$\pi^\pm-$ meson mass exp.	139.57039 ± 0.00018	

Table 4: Masses of full parton models in the generation n = 1.

Parton	mass formula	MeV
$m_P(\pi^0_{4r})$	$(16+(2w_8+u_{16}-3)+4(a_4^2-1))*\frac{1}{2}(w_4^4+1)m_Q$	53.283
$m_{P,em}(\pi^\pm_4)$	$(16+(2w_4+2w_8+u_{16}-5)+2(a_4-1))w_4^4 m_Q$	54.150
$m_{P,0}(\pi^\pm_4)$	$(16+(2w_8+u_{16}-3)+4(a_4-1))w_4^4 m_Q$	54.150
$m_P(\pi^0_{6r})$	$(24+(2w_{12}+u_{24}-3)+2(a_6^2-1))*\frac{1}{2}(w_6^6+1)m_Q$	81.687
$m_{P,em}(\pi^\pm_6)$	$(24+(2w_6+2w_{12}+u_{24}-5)+8(a_6-1))w_6^6 m_Q$	85.423
$m_{P,0}(\pi^\pm_6)$	$(24+(2w_{12}+u_{24}-3)+10(a_6-1))w_6^6 m_Q$	85.423

Table 6a: Mass contributions to models of K^0 - mesons,
(energy equivalents of masses in MeV)

K^0 compon.	Mass contribution (MeV)
ring mass	$2w_8^8 m_Q = 7.558\,988$
skeleton	$64 * 2w_8^8 m_Q = 483.775\,2$
weak ch.	$2(2w_{16}+u_{32}-3)*2w_8^8 m_Q = -0.03\,352$
fluct. ch.	$48(a_8^2-1)*2w_8^8 m_Q = 13.870$
full K^0 model	$2(5+2w_{16}+u_{32}+24a_8^2)*2w_8^8 m_Q$
mod. sum	497.612 MeV
exp. value	497.611 ± 0.013 MeV

Table 6b: Mass contributions per parton to models of K^{*0} - mesons, generation n = 2 (energy equivalents of mass values in MeV)

K^{*0} compon.	Mass contribution (MeV)
ring mass	$2w_{12}^{12}m_Q = 9.133\,184$
skeleton	$96 * 2w_{12}^{12}m_Q = 876.785\,7$
weak ch.	$2(2w_{24} + u_{48} - 3) * 2w_{12}^{12}m_Q = -0.079\,04$
fluct. ch.	$72(a_{12} - 1) * 2w_{12}^{12}m_Q = 18.855$
full model	$2(9 + 2w_{24} + u_{48} + 36a_{12}) * 2w_{12}^{12}m_Q$
mod. value	895.561
exp. value	$895.55 \pm\ 0.20\ \ MeV$

Table 7: Mass contribitions per parton of the model of the $K^{\pm}$ - meson (energy equivalents of masses in MeV)

$K^{\pm}$	parton [1]	(anti)parton [2]	[1] + [2]
skeleton	$32 * 2w_8^8m_Q = 241.887\,6$		483.775
weak charges	$(2w_{16} + u_{32} - 3) * 2w_8^8m_Q$		$-0.033\,6$
value, MeV	$= -1.676 \times 10^{-2}$		
fluct. ident.	$16(a_8^2 - 1) * 2w_8^8m_Q = 4.623\,3$		9.2466
em ; add.fluct.	$(2w_8 - 2) * 2w_8^8m_Q$	$(a_8^2 - 1) * 2w_8^8m_Q$	
value, MeV	$= 0.286\,2$	$= 0.288\,96$	$0.575\,2$
parton sum	246.780	246.783	493.563

The mass of the full $K^{\pm}$ model can be written

$$(2(32 + 2w_{16} + u_{32} - 3 + 16(a_8^2 - 1)) + (2w_8 - 2) + (a_8^2 - 1)) * 2w_8^8m_Q = 493.564\ MeV$$

Table 8: Mass contribitions to the model of the $K^{*\pm}$ - meson (energy equivalents of masses in MeV)

$K^{*\pm}$	parton [1]	antip. [2]	[1] + [2]
skeleton	$48 * 2w_{12}^{12}m_Q = 438.393$		876.786
weak, formula	$(2w_{24} + u_{48} - 3) * 2w_{12}^{12}m_Q$		
weak, value	$-3.952 \times 10^{-2} * 2$		$-0.079\,04$
fluct. ch., form.	$28(a_{12} - 1)) * 2w_{12}^{12}m_Q$		
value	$7.332\,4$		14.665
em charge	$(2w_{12} - 2) * 2w_{12}^{12}m_Q$		
value	$0.523\,8$	0	0.524
parton mass	446.210	445.686	891.896
exp. value	$891.66 \pm\ 0.26\ MeV\ \ (hadroproduced)$		

The mass of the full $K^{*\pm}$ model can be written

$$(2(48 + 2w_{24} + u_{48} - 3 + 28(a_{12} - 1)) + (2w_{12} - 2)) * 2w_{12}^{12}m_Q = 891.896\ \text{MeV}$$

Table 9a: Mass contributions to the model of the J/Ψ -meson
(energy equivalents of masses in MeV)

J/Ψ - comp.	Mass contribution (MeV)
ring mass	$4w_{16}^{16}m_Q = 23.850\,348$
skeleton	$128 * 4w_{16}^{16}m_Q = 3052.845$
weak	$2(2w_{32} + u_{64} - 3) * 4w_{16}^{16}m_Q = -0.308\,646$
fluct.	$2(20(a_{16} - 1) + 2(a_{16}^2 - 1)) * 4w_{16}^{16}m_Q = 44.329\,4$
model value	3096.865
exp. value	$3096.900 \pm\ 0.006\ \ MeV$

The mass of the full J/Ψ model can be written

$$2(64 + 2w_{32} + u_{64} - 3 + 20(a_{16} - 1) + 2(a_{16}^2 - 1)) * 4w_{16}^{16}m_Q$$
$$= 3096.865\ MeV$$

Table 9b: Mass contributions to the model of the $\Upsilon(2S)$ - meson
(energy equivalents of masses in MeV)

$\Upsilon(2S)$−compon.	Mass contribution
ring mass	$4w_{24}^{24}m_Q = 51.547\,811$
skeleton	$192 * 4w_{24}^{24}m_Q = 9897.180$
weak	$2(2w_{48} + u_{96} - 3) * 4w_{24}^{24}m_Q = -0.909\,7$
fluct.	$2(18(b_{24} - 1) + 2(b_{24}^2 - 1)) * 4w_{24}^{24}m_Q = 127.082$
model value	10023.352
exp. value	$10023.26 \pm\ 0.31\ \ MeV$

The mass of the full $\Upsilon(2S)$ model can be written

$$2(96 + (2w_{48} + u_{96} - 3) + 18(b_{24} - 1) + 2(b_{24}^2 - 1)) * 4w_{24}^{24}m_Q$$
$$= 10023.352\ MeV$$

Table 10: Meson models a s examples of particles considered as "tetraquark - candidates".

Electroweak contributions to the parton masses are omitted.

m_{exp}	m_{mod} (MeV)	N_f	l	q. content
$\chi_{c1}(3872)$	$(64+36(a_{16}^2-1))*4w_{16}^{16}m_Q=1594$	72	16	
	$(64+36(a_{20}^2-1))*4w_{20}^{20}m_Q=2277$	72	20	
	$1594+2277=3871$	144		$c\bar{c}q\bar{q}$
$\chi_{c1}(4140)$	$(64+32(a_{18}^2-1))*4w_{18}^{18}m_Q=1876$	64	18	
	$(64+32(a_{20}^2-1))*4w_{20}^{20}m_Q=2264$	64	20	
	$1876+2264=4140$	128		$c\bar{c}s\bar{s}$
$X(4740)$	$(64+42(a_{18}^2-1))*4w_{18}^{18}m_Q=1901$	42	18	
	$(64+44(a_{22}^2-1))*4w_{22}^{22}m_Q=2841$	44	22	
	$1901+2841=4742$	86		$c\bar{c}s\bar{s}$
$X(6900)$	$(64+16(a_{18}^2-1))*4w_{18}^{18}m_Q=1836$	32	18	
	$(96+18(a_{24}^2-1))*4w_{24}^{24}m_Q=5061$	36	24	
	$1836+5061=6897$	68		$c\bar{c}c\bar{c}$
$Z_b(10610)$	$(64+32(a_{20}-1))*4w_{20}^{20}m_Q=2209$	32	20	
	$(96+32(a_{28}-1))*4w_{28}^{28}m_Q=8401$	32	28	
	$2209+8401=10610$	64		$b\bar{b}u\bar{d}$

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