The geometry of magic nuclei in a circular extra space

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Abstract

'Magic' nuclei are defined using a particular energy gap in the separation energy of dineutrons in even – even nuclei. Besides the four small magic numbers 2, 8, 20, 28, the four big magic numbers 50, 82, 126 and 162 are discussed. Each magic number is characterized by a geometry of the nuclear eigenspace with a special symmetry. The symmetry is point like for the magic number 2, and cross - like for the light magic numbers 8, 20 and 28. The big magic numbers have a wing – like geometry with several symmetric 'wings' in three to eight layers. The size of the magic eigenspaces of nuclei are estimated and compared with the measured charge radii of the nuclei in spacetime. The extrapolation of the geometry of eigenspaces into the region of superheavy nuclei up to Z = 126 is discussed.

1 Introduction

BIg differences in the stability of nuclei, a high number of stable isotopes for certain elements, and other unusual nuclear properties were well known during the 1930's (see historical remarks in [3]). However, the physicists did not understand the origin of such differences, they appeared 'magic'. This term was coined by Eugene Wigner. In the 1940's it became clear, that the nuclear shell model could provide a solution. The shell model was known from the electronic shells of atoms. However, one obtained a straightforward solutuion only for the magic numbers up to 20. Only the works of Mayer - Goeppner and in parallel from Jensen showed, that a very strong spin - orbit coupling could provide the correct solution also for the higher magic numbers [2]. Mayer - Goeppner and Jensen worked together and received the Nobel prize in physics 1963.

Based on the shell model, but without the spin - orbit coupling, Pauling proposed a geometric 3D - model for the nuclear structure. He divided nuclei into a core and a mantle, in which groups of nuclei like alpha-particles surrounded the core in closest packing [1].

During the last years, researchers attempted to derive the reasons for the existence of magic nuclei from first principles of nucleon interaction and to leave the assumptions of the strong single-particle - interaction with 'orbitals' by considering special forms of degeneration of the single-particle states. ('pseudo-spin' theories [19]).

In the present article, a geometrical model is proposed which is essentally two-dimensional in its levelled form. Nucleons do not exist as single particles, but only in groups such as (2n2p), (4n2p), (2n) and (2p). These groups are spin-compensated and consequently bosonic. They have no orbitals, and a mean field does not exist in the eigenspaces. The main interaction between (2n2p) - groups is performed through contact points with the nucleons in the neighborhood. The magic numbers can be explained by the special geometric symmetries of the eigenspaces.

The levelled 2D - plots of eigenspaces show compartiments in form of crosses or wings. They are positioned one over the other in the 3D - realization of the eigenspace. A single cross or a wing becomes a layer in the 3D - formation. A filled cross or wing in the eigenspace corresponds a filled shell in spacetime. However, the analogy to the shell model is not complete. In the shell model, all shells already filled are conserved in heavier nuclei. In contrast, the whole structure of a nucleus is completely reorganized from cross-like to wing-like symmetries at the magic number 50. Thus a magic nucleus with N or $Z > 50$ does not contain an 'inert core' of filled structures from the state of a lighter nucleus.

2 The coupling of nucleons

This section represents a summary of the section 'The interaction of nucleons' in a previous article [14]. The models of nucleons follow a dual space concept, they

live in two spaces at the same time. Besides the usual existence in spacetime, the models also exist in an individual 'eigenspace', not embedded in spacetime and fixed to the structure of each nucleus ([13]). The nucleon models show dress - as well as skeleton - coupling in its eigenspaces.

The models allow the formation of a *common eigenspace* of two or three nucleons by radial or axial approach of the *dresses* of the nucleons. A common eigenspace needs less fluctuating charge pairs and therefore less energy than the separate eigenspaces of unbound nucleons. Thus the loss of fluctuating charge pairs represents the main source of the binding energy of the developing nucleus. The loss of fluctuating charge pairs appears as the main mechanism which causes the attracting nuclear force.

A second mechanism concerns the role of the budget of charge clusters in the *skeleton* of the neutron model.

The number of charge clusters per mass quantum may change during the nuclear coupling. One parton of the neutron, the so called 'light arm' (LA), has in the *free* neutron enhanced mass quanta of the type

$m_4 = w_4^4 m_Q = 3.376\,6\ \ MeV$. Each mass quantum is enhanced by the cluster factor $w_4^4 = 1.038\,045$.

In the model of the *bound* neutron the LA parton has only half that number of charge clusters and becomes more similar to the proton model. The average cluster factor becomes $\frac{1}{2}(w_4^4+1) = 1.019\,022$ and leads to enhanced mass quanta with the average mass

$m_{41} = (\frac{1}{2}w_4^4 + \frac{1}{2})m_Q = 3.314\,726\ \ MeV.$

The small mass decrease between m_4 and m_{41} contributes to the binding energy of the neutron in the corresponing bound state like in the deuteron or in an alpha-particle. The reduction of the budget of charge clusters in the *light block* of the neutron represents a second source of attractive nuclear forces.

The mass quanta of the *heavy blocks* of proton and neutron amount to

$m_8 = w_8^8 m_Q = 3.779\,494\ MeV$ in all cases and do not change during the coupling process. The cluster factor is

$w_8^8 = 1.161\,902\,845$.

The skeleton of a bound neutron is absolutely identically with the skeleton of a proton. This is possibly the reason for the stability of the bound neutron in contrast to the unstable free neutron.

The coupling of rotons by radial or axial approach The loss of fluctuating charge pairs during the formation of a common eigenspace represents a special type of the nuclear coupling, the coupling *between* different nucleons. The nuclear force cannot be understood as a 'residuum' of the strong force, the coupling between the partons *within* a nucleon. This assumption extends the number of fundamental forces.

Two rotons belonging to different nucleons need to have parallel spin axes in order to get coupled. Two different geometries of coupling are possible:

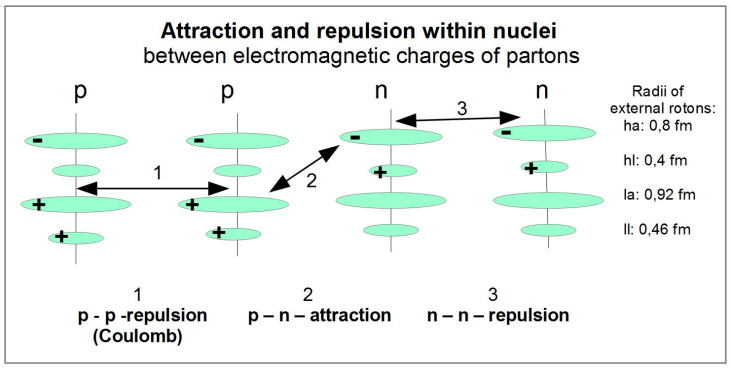


Figure 1: Types of electromagnetic interaction between protons and neutrons within a common eigenspace. The p - p - interaction is dominated by repulsion, known from classical Coulomb - interaction. The n -p - attraction and the n - n - repulsion are non-classical interaction types. The actual total (np) - interaction is a mix of different types, which can compensate each other.

- Radial coupling of spin-antiparallel rotons. The spin axes are parallel, however they have always a distance of the order of the two radii of the rotons. The coupled nucleons exist in the same layer. Axial couplig represents the main coupling method.
- Axial coupling of spin-parallel rotons. The spin axes become identically. The coupled nucleons exist in different layers. Axial coupling represents a rare coupling method.

In both cases, the rotons generate a common circular eigenspace with a reduced number of fluctuating charge pairs. The common eigenspace is two-dimensional perpendicular to the spin axis.

The reduction of the number of fluctuating charge pairs reduces the total rotating energy of the coupled system, the difference appears as binding energy of the system.

The interaction between charged partons within a common eigenspace of bound nucleons comprises the classical p - p - repulsion and several non-classical variants such as p - n - attraction and n - n - repulsion, see Fig. 1.

3 Magic nuclei

3.1 The alpha-particle - the smallest magic nucleus

The alpha-particle has an extrordinary stability, therefore the nucleus 4 He represents the first, lightest doubly magic nucleus. In a common eigenspace of an even - even nucleus, the spin-compensated group (2n2p) = $(n \uparrow n \downarrow p \uparrow p \downarrow)$ plays a leading role. The reason is the favorable possibility to set free a relatively

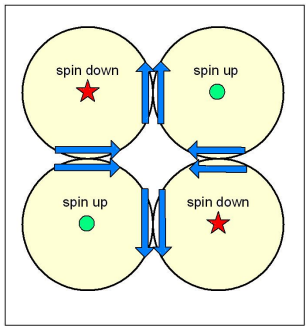


Figure 2: Rotons of four nucleons bound in an alpha-particle in attractive contact-interaction with two antiparallel coupling partners per nucleon. The coupling in the alpha-particle generates a ring-like chain of attraction.

high number of fluctuating charge pairs. The attractive interaction between a nucleon and its two neighbors in a (2n2p) - group with ant-parallel spins can be illustrated using a 'cogwheel-picture', see Fig. 2. Four selected rotons, one roton per nucleon, can be considered to build a ring-like construction of four cogwheels rotating with the velocity of light. The four contact points between the four rotons are depicted as double arrows. At the contact points, the peripheral movements of rotating masses and charges are equi-directional.

From the mechanical picture one would suppose, that equal diameters - caused by equal masses of the rotons - are a good condition for the coupling. The tendency to achieve equal masses is called 'synchronization'.

The alpha - model with synchronized partons of the four nucleons can give numerical results, in particular the binding energy of the alpha-particle, see [14]. The main parameters of skeletons and dresses of proton and neutron in a free state and bound in an alpha-particle are given in Fig. 3. The nearly perfect synchronization of the bound proton and neutron can be seen by comparing the total masses of the models of proton and neutron bound in the alpha - particle (lower display, red backgraund color).

The symbols H and L mean the heavy and the light block of the skeleton masses, N_f means the number of fluctuating charges in the dresses of the models. We use the abbreviation $m_k = w_k^k m_Q$, where m_Q means the mass quantum and w_k^k the energy factor of a rotating charge, see the appendix.

The model masses of neutron and proton bound in the alpha-particle provide the contribution of one nucleon to the binding energy:

$B(n_\alpha) = 939.565\,367 - 931.852\,648 = 7.712\,719\ MeV$
$B(p_\alpha) = 938.272\,129 - 931.837\,125 = 6.435\,004\ MeV$

The sum of both contributions represents an important constant in calculations of the binding energy of light nuclei up to Z = 28. The binding energy of a (np) - pair, where each nucleon has two coupling partners, amounts to

n free	N_f	skeleton
H ⊖ ⊕	162	$160m_8$
L	96	$32m_4 + 64m_{41}$
$n \equiv n^0$	258	$256m_Q$
$m_{n,\mathrm{mod}}$	939. 565 367 MeV	

p free	N_f	skeleton
H ⊖	174	$160m_8$
L ⊕ ⊕	92	$96m_{41}$
$p \equiv p^+$	266	$256m_Q$
$m_{p,\mathrm{mod}}$	938. 272 129 MeV	

n in α	$N_f(\alpha)$	ΔN_f	m_{skel}	Δm_{skel}
H ⊖ ⊕	104	58	$160m_8$	0
L	48	48	$96m_{41}$	$32(m_4 - m_{41})$
$n \equiv n^0$	152	106	$256m_Q$	$32(m_4 - m_{41})$
$m_{n,\alpha}$	931. 852 648			MeV

p in α	$N_f(\alpha)$	ΔN_f	m_{skel}	Δm_{skel}
H ⊖	104	70	$160m_8$	0
L ⊕ ⊕	44	48	$96m_{41}$	0
$p \equiv p^+$	148	118	$256m_Q$	0
$m_{p,\alpha}$	931. 837 125			MeV

Figure 3: Skeleton and dress parameters of the models of neutron and proton in its free state and bound in an alpha-particle.

$$B(np)_2 = B(n_\alpha) + B(p_\alpha) = 14.\,147\,723\ MeV \tag{1}$$

Two such (np) - pairs show an internal compensation of attractive and repulsive electromagnetic forces. They form the lightest 'doubly magic' nucleus with N = Z = 2. Two (np) - pairs appear as building stones of different even - even light nuclei and contribute a *constant* binding energy, see [14], Table 4.

The binding energy of the alpha-particle becomes

$$B_{\alpha,\mathrm{mod}} = 2B(np)_2 = 28.\,295\,446\ MeV \tag{2}$$

This result has a deviation of $7 * 10^{-6}$ from the experimemtal value [1]).

$B_{\alpha,\mathrm{exp}} = 28.\,295\,639\ MeV$.

If different (2n2p) - groups couple together in a nucleus, one (np) within a (2n2p) - group gets third partner-nucleons for the nuclear coupling. The binding energy for one (np) increases by B_{32}, the binding energy of the contact point *between* two (2n2p) - pairs. The total binding energy of a (np) with athird partner of each nucleon becomes

$$B(np)_3 = B(np)_2 + B_{32;O} = 17.765273\ MeV \tag{3}$$

The binding energy B_{32} of a contact point depends on Z, the empirical value $B_{32,O} = 3.\,617\,55\ MeV$, calculated from O 16 data, is approximately valid for even - even nuclei up to Z = 50 ([14]).

The binding energy of a dineutron depends on the position within the nucleus and also on the number of (2n2p) -pairs and dineutrons already present in the nucleus. The attraction of a dineutron increases with an increasing number of (2n2p) -groups. Attraction means increase of binding energy due to *loss* of fluctuating charge pairs. The repulsion of a dineutron increases with increasing

[1] Experimental values on particle masses as well as masses and binding energies of nuclei are taken or calculated from CODATA [9] and AME [4].

number of other dineutrons already present in the nucleus. Repulsion means decrease of binding energy due to the *gain* of fluctuating charge pairs.

The binding energy of a dineutron in a neutral state, without additional attractive or repulsive interaction from outside the group, amounts to

$$B(nn)_0 = 2B(n_\alpha) = 2 * 7.712\,719 = 15.425\,44\ MeV \tag{4}$$

The measured separation energy S(2n) varies in a wide range from a few MeV to 28 MeV.

3.2 The identification of magic nuclei by energy gaps

The usual criteria to identify a magic nucleus are its extraordinary high stability, and a high number of stable nuclides in the isotopic branch belonging to the magic number. These criteria work well for light and medium heavy atomic numbers up to Z = 82. In the region of super heavy nuclei it seems to be useful to apply in addition different criteria.

In this article, we use the energy gaps in the curves of the separation energy S(2n) in dependence on the atomic mass A. For the Z region 2 to 56 it is shown in Fig. 4, for Z $>$ 50 in Fig. 5. The data in the figures are calculated from [4].

The nucleus 14 C has a separation energy of the first dineutron of 13,12 MeV. The first dineutron of 18 O can be separated with 12,19 MeV. It is bound with somewhat less energy, because 16 O is more symmetric than 12 C and the addition of a dineutron is energetically unfavorable. The nucleus 16 O represents the next 'doubly magic nucleus' with N = Z = 8.

The highest separation energy S(2n) $\approx$ 28.2 MeV is found at Z = 48 (Cd), followed by the greatest gap of 4.4 MeV down to $S(2n) \approx 23.8\ MeV$ at the 'magic' number Z = 50 (Sn). That means, the doubly magic nucleus 100 Sn (with N = Z = 50) has a structure of a special high symmetry, where a dineutron can be added with a reduced binding energy only. The symmetric geometry of 100 Sn (or another magic nucleus) has only unfavorable positions for additional bosonic groups.

At higher atomic numbers we find magic numbers at 50, 82, 126 and 162. The magic number 162 has been derived from the 'separation energy gap' in the region Z = 102 to 110. This represents a relatively small amount of information. The 'gap' can be observed at four even - even nuclei only, namely 266 Rf (Z = 104), 268 Sg, 270 Hs, and 272 Ds (Z = 110). The representation of the data over the neutron number N will be give a better impression, see Fig. 9.

The isotopic branches over N show identical gaps of the separation energy S(2n), see Figs. 6 to 10.

Doubly magic nuclei and its isotopic and isotonic branches The first and most famous magic number is two, Z = N = 2 means the alpha-particle. It is stable and has an extraordinary high binding energy. The next three numbers are eight, 20, and 28. These four light magic numbers 2, 8, 20, and 28 are characterized by a doubly magic nucleus with N = Z at a crossing point of a

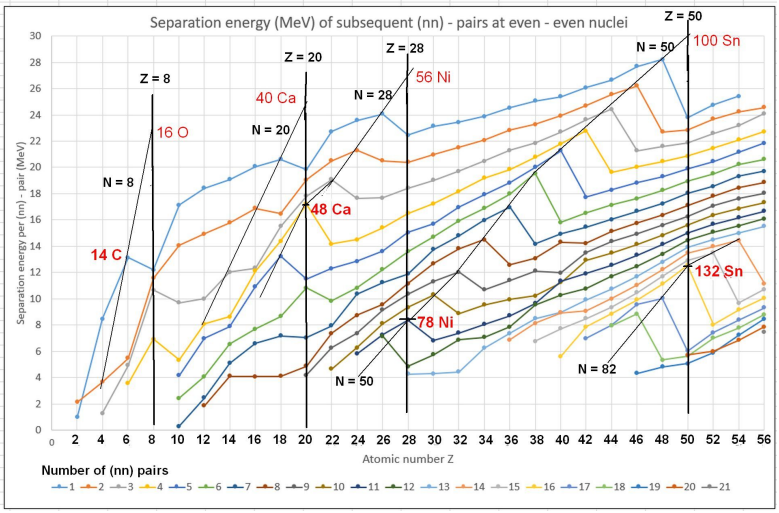


Figure 4: Identification of the magic numbers 8, 20, 28, and 50 by characteristic gaps in the separation energies S(nn) of subsequent dineutrons. At the magic atomic numbers Z = M_{mag}, the first dineutron has a significantly lower separation energy compared to the neighbor nucleus with the atomic number $Z = M_{mag} - 2$.

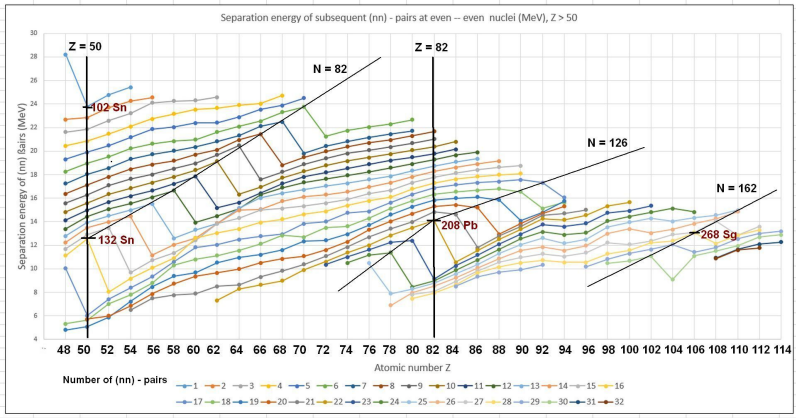


Figure 5: Identification of the magic numbers 50, 82, 126, and 162 by characterisic gaps of the separation energies of dineutrons attached to even - even nuclei.

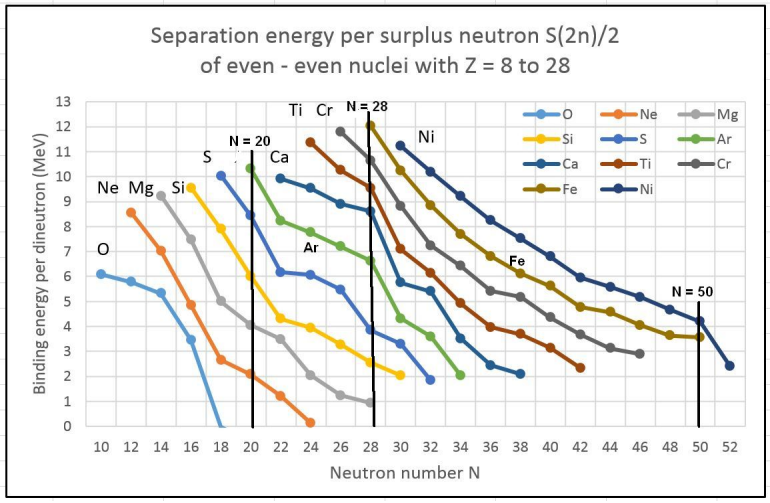


Figure 6: Identification of the magic neutron numbers N = 20, 28, and 50 by gaps in the separation energy S(2n)/2

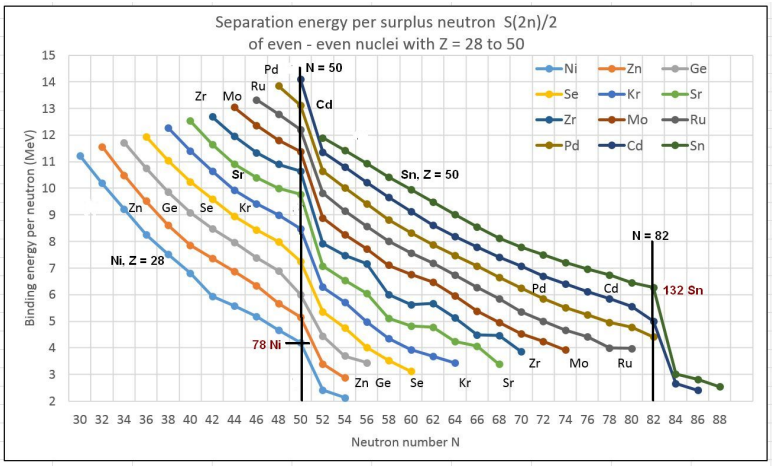


Figure 7: Identification of the magic neutron numbers N = 50 and N = 82 by gaps in the separation energy S(2n)/2.

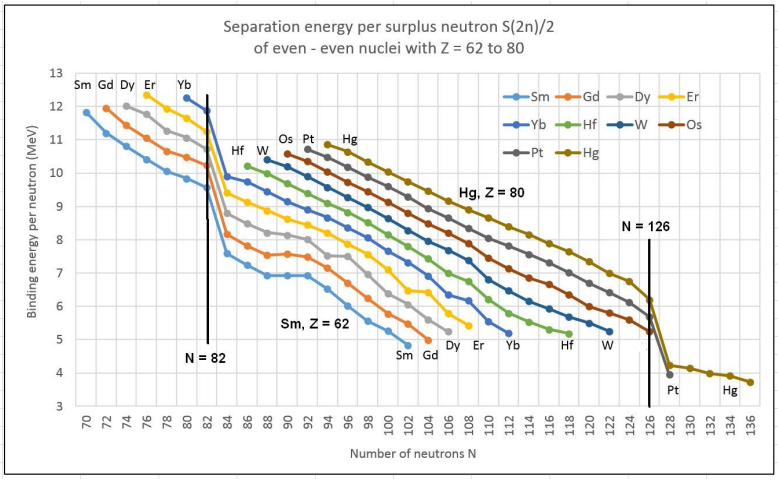


Figure 8: Identification of the magic neutron numbers N = 82 and N = 126

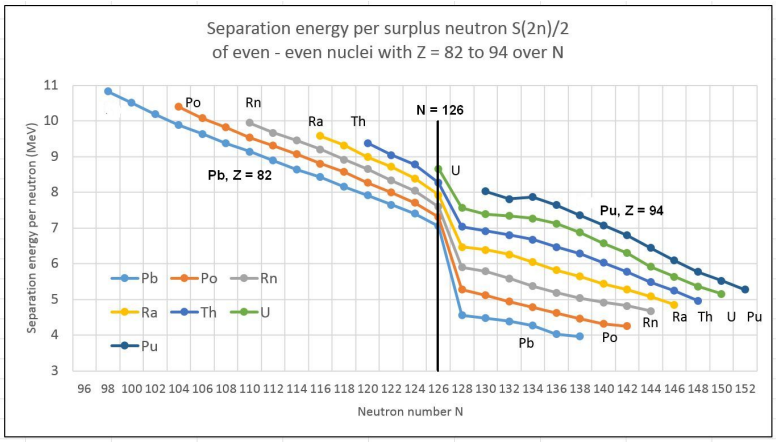


Figure 9: Identification of the magic neutron nuimber N = 126 by gaps in the separation energy S(2n).

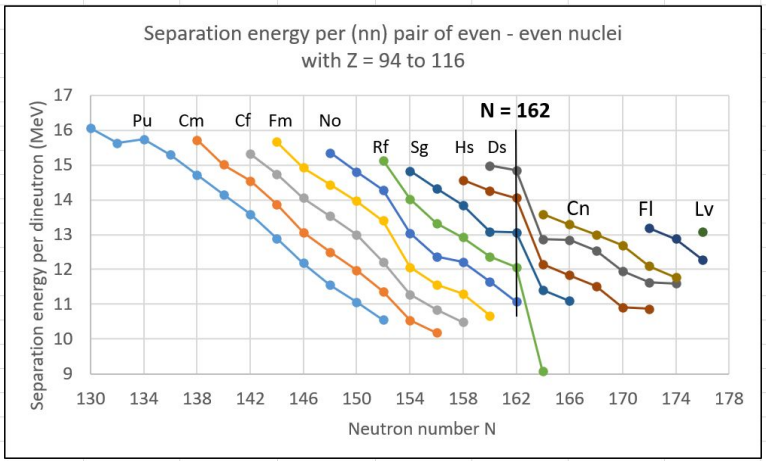


Figure 10: Identification of the magic neutron number N = 162 by gaps in the separation energy S(2n)/2.

branch of isotopic magic nuclei (with identical magic Z) and a branch of isotonic magic nuclei (with identical magic N), see Fig. 11.

Heavy magic nuclei have the magic numbers 50, 82, 126 and 162. A doubly magic nucleus is realized in nature only for N = Z = 50. The magic number 82 is realized in form of the isotopic and isotonic branches. The magic numbers 126 and 162 can be seen from small pieces of its isotonic branches only, see Fig. 12.

For of the magic number 162 we have only five points of the isotonic branch, the magic nuclei 264 No, 266 Rf, 268 Sg, 270 Hs, and 272 Ds. They have N = 162. Nuclei with the atomic number Z = 162 probably do not exist.

3.3 The seat concept for the eigenspaces of magic nuclei

The smooth curves in the Figs. 11 and 12 lead to the presumption, that a certain geometric property could be the reason of the similar binding energies within a certain isotopic or isotonic branch. This idea is the background for a 'seat concept', a kind of geometric lattice of 'placeholders' common to all nuclei of both branches with the same magic number. The difference between two nuclei with identical seat configuration could exist exclusively in the type of structural components sitting on the seats. The structural components in question are listed in Fig. 13. The seat concept will be discussed in the next subsections.

3.3.1 Light magic nuclei

As an example for $M_{mag} = 8$, we fill the four seats with different structural components (see Fig. 13).

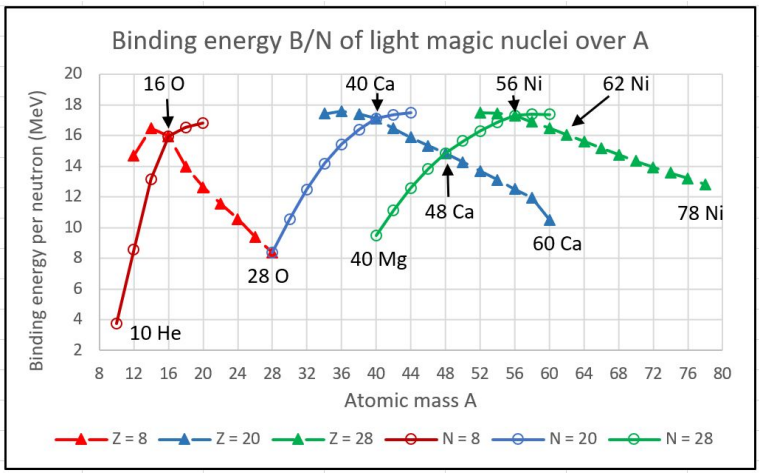


Figure 11: The binding energy per neutron has isotopic and isotonic branches beginning at the crossing points N = Z = 8, 20, and 28.

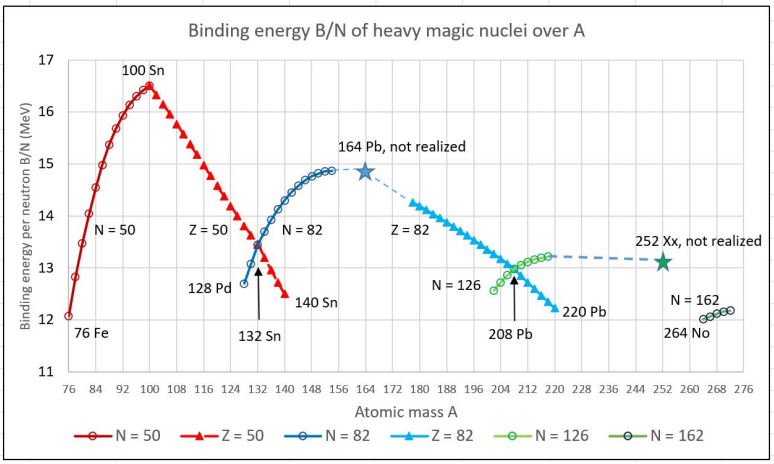


Figure 12: The binding energy per neutron has isotopic and isotonic branches of the magic numbers 50, 82, 126, and 162. The isotopic branches with Z = 126 and 162 are missing, only fractions of the isotonic branches are realized.

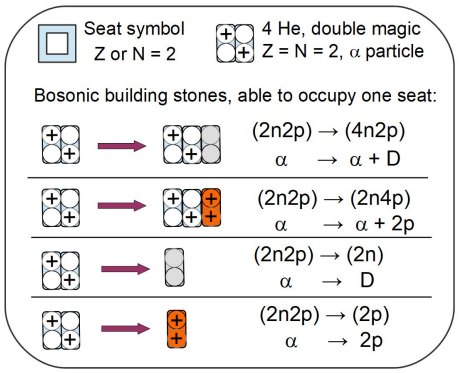


Figure 13: 'Magic' configuration of one seat and groups of nucleons able to occupy one seat. Four possibilities to complement the block (2n2p) or to replace it represent the development of two isotopic and two isotonic branches extending from a nucleus with N = Z.

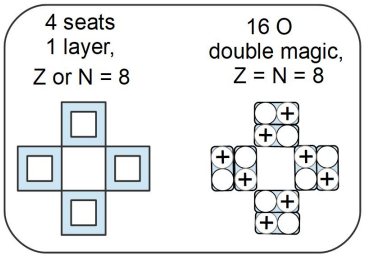


Figure 14: 'Magic' seat configuration with four seats in one layer as 2D - plot (at the left). In the model of 16 O (at the right), four (2n2p) - groups occupy four seats.

Taking the nucleus 16 O as crossing point (14), two branches have atomic masses A > 16:

- the upper isotonic branch with 18 Ne and 20 Mg, having N = 8, and
- the upper isotopic branch with 18 O, 20 O up to 28 O, having Z = 8.

Two branches beginning at 16 O have atomic masses A < 16:

- the lower isotonic branch with 14 C, 12 Be and 10 He, having N = 8, and
- the lower isotopic branch with 14 O, 12 O having Z = 8.

From the nucleus 16 O as doubly magic nucleus develop four branches, two containing isotonic (see Fig. 15) and two containing isotopic nuclei (see Fig. 16). These 'simple magic nuclei' have a similar cross-like geometry of its eigenspace

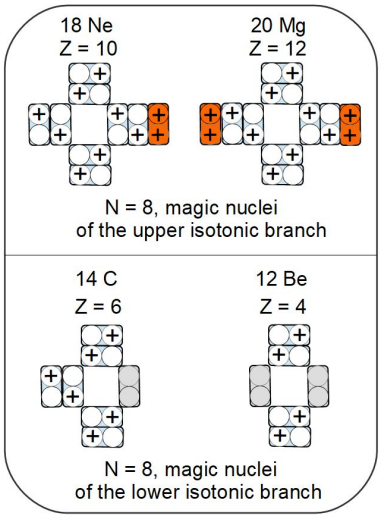


Figure 15: Two isotonic branches (with N = 8) start from 16 O. The upper isotonic branch (with A > 16) includes 18 Ne; the lower isotonic branch (with A < 16) contains 14 C.

Any bosonic group occupying one seat can have different positions in space-time. The seat configuration cannot be interpreted as a picture of the nuclear structure in spacetime.

Fig. 17 shows two magic seat configurations with 10 and 14 seats in two and three layers as 2D - plots. The upper panel belongs to 40 Ca as well as to the magic nuclei of the isotonic and the isotopic branches. The lower panel chracterizes the nuclide 56 Ni and the two branches emerging at this crossing point.

A schematic presentation of the model of 40 Ca in three dimensions is given in Fig. 18.

The binding energy of magic nuclei near to the doubly magic nucleus 48 Ca is shown in Fig. 19. The slopes of the curves change at A = 48, indicating a structural change from the two-layer to a three-layer configuration. Both configutations have identical binding energies at the transition point A = 48.

At the atomic mass $A \leq 48$,the average slope is 18.5 MeV per dineutron (17.2 to 19.8 MeV/(2n) between 40 Ca and 48 Ca), that is before the energy gap.

However, at $A \geq 48$ (beyond the energy gap) the average slope becomes 8.6 Mev per dineutron (4.8 to 11.5 MeV/(2n) between 48 Ca and 56 Ca). Such a drastic, permanent change of the slopes can be explained by a transition into a new geometry of the eigenspace. We assume a geometric change with two crosses in two layers for atomic masses below 48 and three crosses in three layers for atomic masses above 48, with A = 48 as the transition point. The transition point is defined by an equal binding energy in both geometries.The geometric

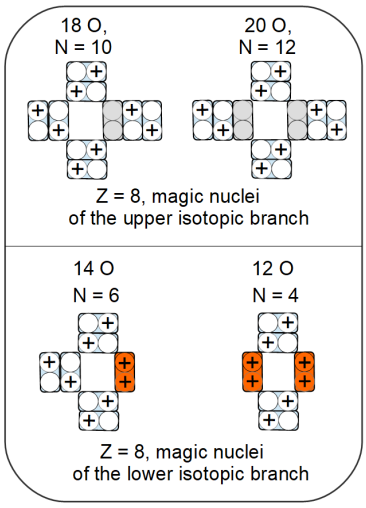


Figure 16: The upper and lower isotopic branches include the oxygen - isotopes (12 O to 20 O, Z = 8).

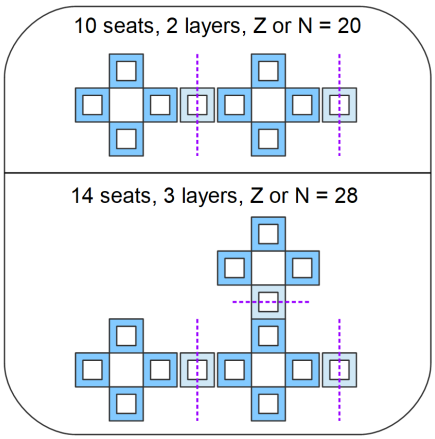


Figure 17: Seat configurations with 10 and 14 seats as 2D - plots. If one seat (colored square) is occupied by a (2n2p) - group, it means 40 Ca (upper panel) and 56 Ni (lower panel). Two crosses have C = 16 contact points (upper display), three crosses have C = 22 contact points.

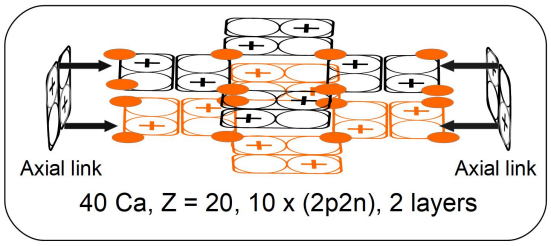


Figure 18: View of the eigenspace of 40 Ca in three dimensions. The two layers have the colors black and orange and are connected by two (2n2p) - groups in axial coupling (schematic view). The number of contact points is 8 per layer, C = 16 in total (red dots).

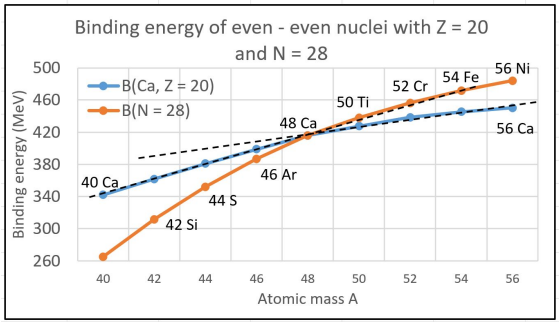


Figure 19: The binding energy of magic nuclei in the neighborhood of the nucleus 48 Ca. The doubly magic nucleus 48 Ca represents the transition point between two possible seat configurations. This indicates the change of the slopes of the curves.

change exists in addition to the increasing repulsion between a growing number of dineutrons, which would occur also within a persistent geometry.

The change of geometries is illustrated using a comparison of the doubly magic mirror nuclei 48 Ca and 48 Ni, an extremely proton-rich nucleus first produced in 1999 [16]. Both mirror nuclei 48 Ca (Z=20, N=28) and 48 Ni (Z=28, N=20) represent a transition point from the 10 seat to the 14 seat configuration, see Fig. 20.

The configuration with 14 seats can be occupied by 11 dineutrons in maximum. Fig. 21 shows a 2D - plot of the eigenspace of 78 Ni. The model consists of 11 (4n2p) groups and three (2n2p) groups acting as links between the three layers. More than 11 dineutrons cannot be positioned on 14 seats.

The slopes of the curves (see Fig. 22) change at A = 78. This indicates a structural change from cross - like to wing - like symmetric geometries.

At $A \leq 78$, the average slope is 10.2 MeV per dineutron. Before the energy gap, the geometry belongs to $M_{mag} = 28$.

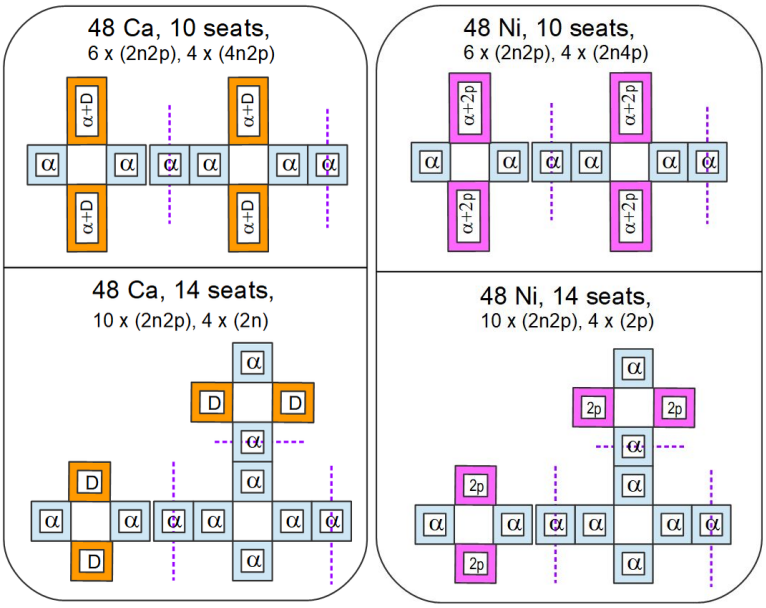


Figure 20: The doubly magic nucleus 48 Ca and its mirror nucleus 48 Ni are at the transition point from the 10 seat to the 14 seat configuration.

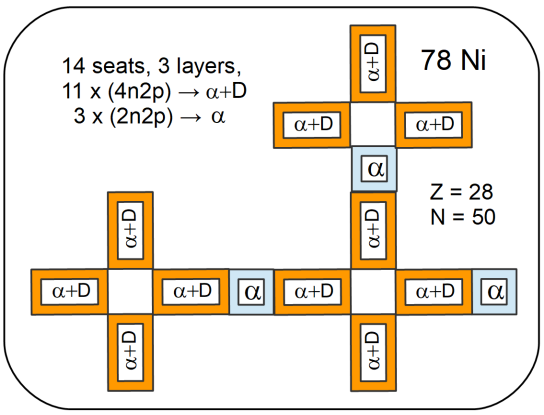


Figure 21: A model of 78 Ni, it contains D = 11 dineutrons, the maximum number for 14 seats. The symbol α means a (2n2p) - group, $\alpha + D$ means (4n2p).

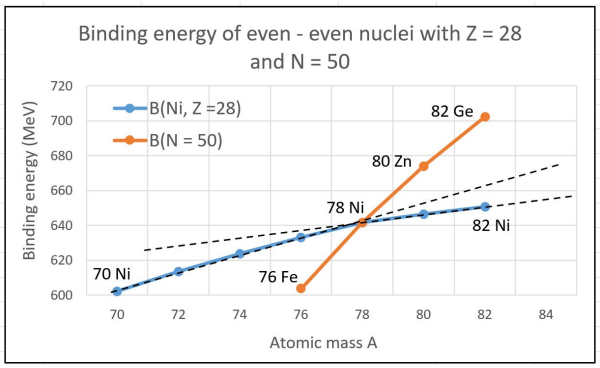


Figure 22: Binding energies of magic nuclei in the neighborhood of the doubly magic nucleus 78 Ni. The change of the slopes indicates the change of the geometry of the eigenspace.

However at $A \geq 78$ the average slope becomes 4.6 MeV per dineutron in the region 78 Ni to 82 Ni. Beyond the energy gap, the geometry belongs to $M_{mag} = 50$. The permanent change of the slopes indicates a transition of the eigenspaces of Ni - nuclei from the cross-like to the wing-like geometry. Thus wings occur already at Z = 28, N = 50.

An eigenspace of the 78 Ni nucleus with wing - like geometry and identical binding energy is shown in Fig, 27. The change to eigenspaces with a wing-like geometry will be discussed in the section on heavy nuclei.

Estimation of the size of models of light nuclei The measured radii of nuclei refer in most cases to the root-mean-square (RMS) charge radius, see [11]. We need the diameters for the comparison with the size of models. Table 1 contains the observed charge radii and diameters of light magic nuclei.

Table 1: Charge radii of light magic nuclei

nuclide	charge radius (fm)	diam. (fm)
16 O	2,6991	5,3982
40 Ca	3,4776	6,9552
58 Ni	3,7757	7,5514

The size of models depends on the size of (2n2p) - groups. The general shapes of the eigenspaces of light nuclei are crosses in one, two or three layers. The geometry of each layer is given as 2D - plot (see Figs. 14, 17, and **??**), that means as a 'flat' schematic representation on a plane. The actual realization of the eigenspace may be more complex. For instance, the 2D-plot of the model of 16 O consists of a 'flat' cross of two bars of the length of two (2n2p) - groups, divided into two arms with a gap between them. The gap is about $2r_1$ wide, where r_1 means the radius of the external roton of the partons HA or LA of a nucleon.

However, the eigenspace of 16 O could consist of two compact bars, having exactly the length of two (2n2p) - groups and overlapping at the crossing point. The model could be non-flat, and difficult to draw on a flat peace of paper. Some contact points could be changed from radial to axial coupling, possibly without changing the energy of the contact.

Therefore, we compare in Table 2 the sizes of cross - like layers by calculating the lengths of one bar consisting of two (2n2p) - groups with and without a gap, that means for the flat and the non-flat variant of the layer.

Table 2: Estimated sizes of cross-like eigenspaces of light nuclei

parton	bar length in a cross (fm)	
	with gap (flat)	overlap (non-flat)
HA (m_8)	8,04	6,432
LA (m_{41})	9,27	7,416

A comparison between the tables 6 and 7 shows, that a certain degree of overlap seems to be a plausible assumption. However we note, that there is no complete theory of translating the length of eigenspace - structures into diameters in spacetime.

Limitations of the seat concept Some light elements have nuclei with an extraordinary high number of dineutrons along its isotopic branch, see Fig. 23. The maximum number of dineutrons exceeds the number of (2n2p) - groups (e.g. alpha-particles) by 2 at Z = 2 to 12:

$$D = Exmax = N(2n2p) + 2, \quad Z = 2...12 \tag{5}$$

This does not allow the application of a unified seat concept for all isotopes. A unified seat configuration requires

$$D \leq N(2n2p); \quad N \leq 2Z \tag{6}$$

which excludes a few light nuclei with a high number of dineutrons.

The structural solution for nuclei with N > 2Z is the split of a (2n2p) - group into 2 (np) groups and to build (3np) - groups, which resembles the situation at Be - isotopes (Fig. 24).

Oxygen - isotopes with D = 0 and with D = 6 have different seat configurations. A unified magic seat concept at Z = 8 is applicable only for D = 0 (N = 8) to D = 4 (N = 16). Higher N, in particular the maximum number D = 6 (N = 20) require a new seat configuration similar to that of Be - isotopes. This does not represent a 'magic' seat configuration. Consequently, the nucleus 28 O is not considered to be a doubly magic nucleus, despite the combination Z = 8 and N = 20 ([17]).

Also non - magic even - even nuclei with Z = 6, and Z = 10 to 14 show a similar situation at high values of D.

Nuclei heavier than the element Argon (Z = 18) have a smaller maximum number D of dineutrons, such as

$D = Exmax \leq N(2n2p) - 1$

and the seat concept is applicable without restrictions (see Fig. 25).

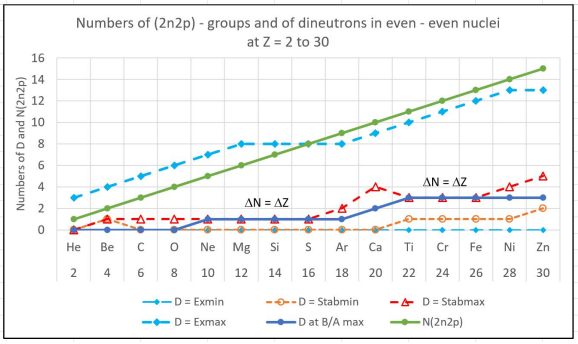


Figure 23: Numbers of dineutrons in nuclei with Z = 2 to 30. The range of realized nuclei has a lower limit at D = Exmin and an upper limit at D = Exmax. The range of stable nuclei reaches from Stabmin to Stabmax. The number D at the highest binding energy per nucleon (B/A) is shown as bold blue curve.

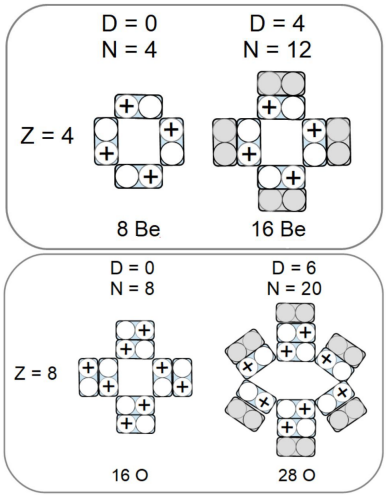


Figure 24: Eigenspaces of Be - and O - isotopes with extremes of the dineutron - content. Very neutron - rich nuclei such as 28 O show a splitting of (2n2p) groups and differ from the four - seat - concept. 28 O is not consdered to be a 'doubly magic' nucleus, despite the parameters (Z, N) = (8, 20).

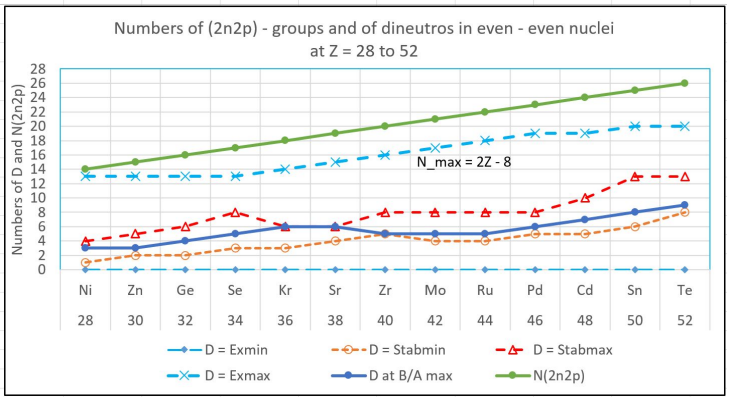


Figure 25: Numbers of dineutrons in existing and in stable nuclei at Z = 28 to 52. The range of realized nuclei has a lower limit at D = Exmin and an upper limit at D = Exmax. The range of stable nuclei reaches from Stabmin to Stabmax. The number D at the highest binding energy per nucleon (B/A) is shown as bold blue curve.

3.3.2 Heavy magic nuclei

The number of dineutrons D does not change with Z over wide distances of Z in stable *light* nuclei (Z = 2 to 50).

In contrast, ΔD increases with Z in stable *heavy* nuclei (Z = 50 to 82). In the average one finds the slope

$\Delta D = (3/8)\Delta Z$ or $\Delta N = (7/4)\Delta Z$, see Fig. 26.

The maximum slope $\Delta N = 2\Delta Z$, barely higher than $(7/4)\Delta Z$, means that the nucleon content of nuclei increases in steps of (4n2p) on different paths through the valley of stability.

The seat concept is applicable without restrictions for magic nuclei with Z > 50, and the group (4n2p) represents the most prominent group occupying the seats. The geometry of seat configurations change from the cross - dominated structures of light magic nuclei (N or Z equals 28 or below) to wing - dominated structures of heavy magic nuclei (N or Z equals 50 or above). This behavior differs fundamentally from the concept of shells, where special 'filled' shells remain present in all nuclei with a higher number of nucleons.

The transition from a cross - like to a wing-like symmetry begins already at 78 Ni, where the magic number N = 50 is reached the first time. The number 50 allows to realize the 'two - wing - geometry' with 25 seats and three layers (see Fig. 27).

The two wings have eight seats each, attached to a 'platform' with 3 x 3 seats. This configuration is shown in Fig. 28. The 2D - plot of the eigenspace of 116 Sn is given as an example, see 29.

We remark, that the 2D - plots of eigenstates are schematic representations on a plane. 2D - plots do not reproduce the size of the nuclei in space-time in

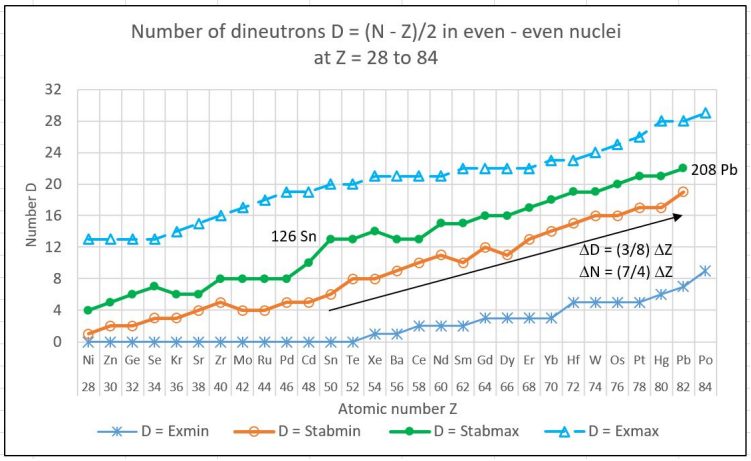


Figure 26: Numbers of dineutrons in existing and in stable nuclei with Z = 28 to 84. The range of realized nuclei has a lower limit at D = Exmin and an upper limit at D = Exmax. The range of stable nuclei reaches from Stabmin to Stabmax.

a 1 : 1 manner.

The binding energies of magic nuclei near to the doubly magic nucleus 132 Sn is shown in Fig. 31. The slopes of the curves change at A = 132, indicating a geometric change of the eigenspace.

At $A \leq 132$ the average slope is 13.5 MeV/(2n), the slope ranges from 12.6 to 14.4 MeV per dineutron.

Behind the energy gap, at $A \geq 132$, the average slope amounts to 5.6 MeV/(2n). The energy gap with a permanent change of the slopes marks the transition into a new geometry of the eigenspace.

One could imagine a seat configuration with 25 + 8 = 33 seats as the next

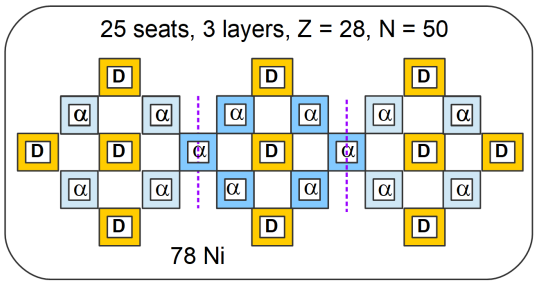


Figure 27: Eigenspace (2D - plot) of the doubly magic nucleus 78 Ni, here with a two - wing - geometry (25 seats, N = 50). The symbol α means the group (2n2p), D means a dineutron (2n).

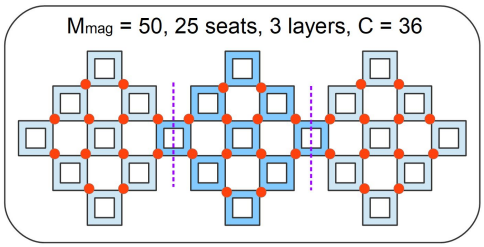


Figure 28: Magic seat configuration with 25 seats in three layers as 2D - plot. Two 8 seat - wings are attached to a 9 seat - platform. Each layer has 12 contact points, C = 36 contacts in total (red dots).

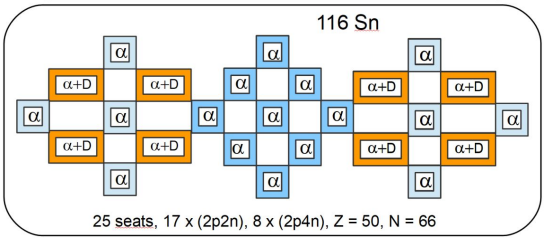


Figure 29: Eigenspace of the nucleus 116 Sn in a configuration with 25 seats in three layers as 2D - plot. Each layer has 12 contact points, C = 36 contacts in total. The model of 116 Sn is given in the lower panel. The symbol α means (2n2p), the synbol α + D means a (4n2p) - group.

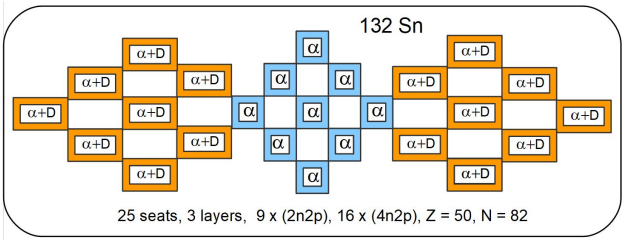


Figure 30: Eigenspace (2D - plot) of the doubly magic nucleus 132 Sn. The number of dineutrons is at its maximum, regarding 25 seats.

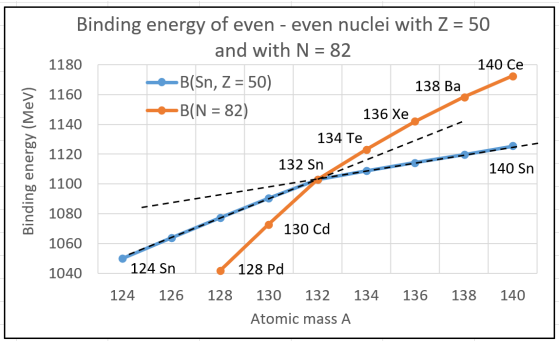


Figure 31: Binding energies of magic nuclei in the heiborhood of the doubly magic nucleus 132 Sn. The change of the slopes at 132 Sn indicates the change from a two-wing geometry (belonging to $M_{mag} = 50$) to a four-wing geometry (belonging to $M_{mag} = 82$). Both geometries have the same binding energy at A = 132.

magic seat configuration with 25 + 8 = 33 seats. However, we could not identify magic properties of nuclei with N or Z = 66. Instead, we found the next magic number at 82, corresponding to 25 + 2*8 = 41 seats. The configuration with 4 wings has an aesthetic symmetry, see Fig. 32. The eigenspace of 132 Sn has been chosen as an example, see Fig. 33.

Fig. 33 shows the 41 seat - configuration in its 'lightest' form (A = 132). The heaviest realization of the configuration with 41 seats seems to be the model of 208 Pb, see Fig. 34.

The configuration with 81 seats in 5 layers is compact and has a relatively high binding energy. However, it cannot accommodate more than 22 dineutrons.

The magic number 162 produces a crossing point between isotonic and isotopic branch far outside the experimental reach. The lower isotonic branch of this magic number has a few members already discovered.

Fig. 37 shows the binding energies of the magic nuclei near to the doubly magic nucleus 208 Pb. The slopes of the curves change at A = 208, indicating a geometric change of the eigenspaces from the four-wing to the five-wing variant of the eigenspace.

At $A \leq 208$ the average slope amounts to 15.3 MeV per dineutron (range: 14.1 to 16.3 MeV/(2n)),

at $A \geq 208$ the average slope becomes 8.6 MeV per dineutron (range: 7.9 to 9.1 MeV/(2n)). This energy gap with a permanent change of the slope indicates the transition from a four-wing to a five-wing geometry of the eigenspaces of the Pb - nuclei.

The 2D - plot of the eigenspace of Darmstadtium 272 Ds with Z = 110 and N = 162 is depicted in Fig. 38. The nucleus of 272 Ds represents one of the heaviest magic nuclei observed in nature.

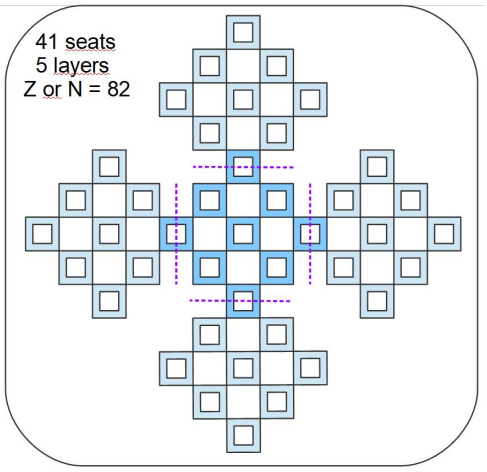


Figure 32: Magic seat configuration with 41 seats in 5 layers as 2D - plot. It has a platform with 9 seats and four wings. Each layer has 12 contact points, C = 60 contacts in total.

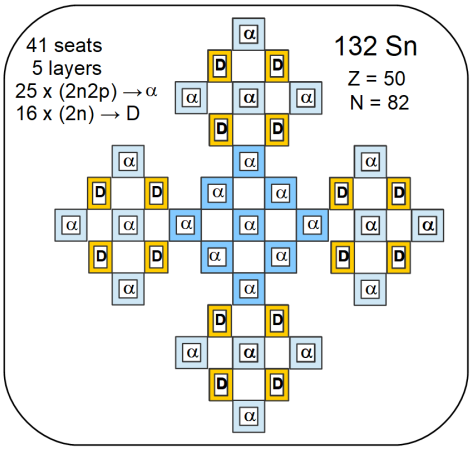


Figure 33: Eigenspace (2D - plot) of the nucleus 132 Sn in an 41 seat - configuration

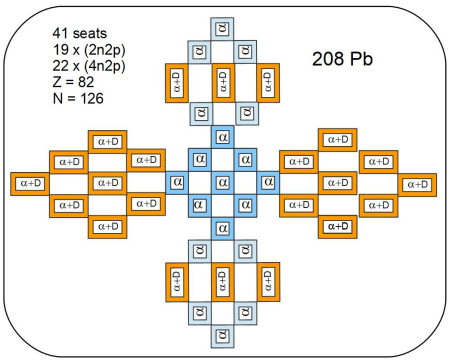


Figure 34: Eigenspace of the 208 Pb nucleus as 2D - plot. The model has 41 seats with four wings attached to a platform, five layers in total. The symbol α means (2n2p), the symbol α+ D means a (4n2p) - group.

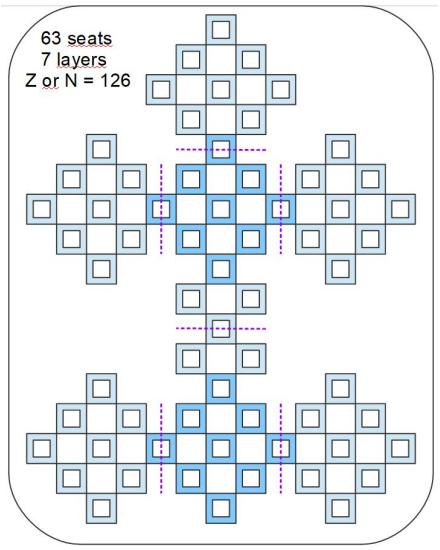


Figure 35: Magic seat configuration (2D - plot) with 63 seats in seven layers. It has two 9 seat - platforms, connected with a link of five seats, and five wings with eight seats each. A wing has 12 contact points, a platform plus link has 16 points, in total exist 92 contact points.

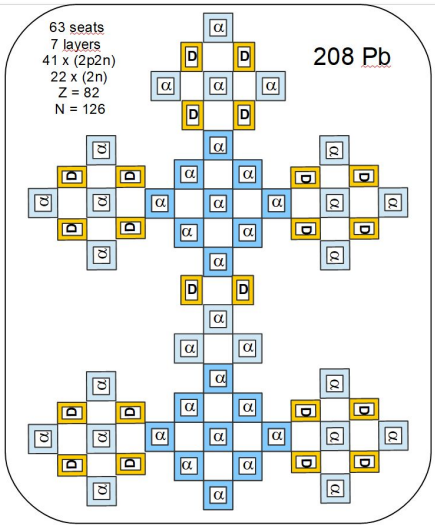


Figure 36: Eigenspace of 208 Pb with 63 seats in 5 layers as 2D - plot. This configuration belongs to the magic number 126, it has space for a higher number of dineutrons than the 41 seats - configuration.

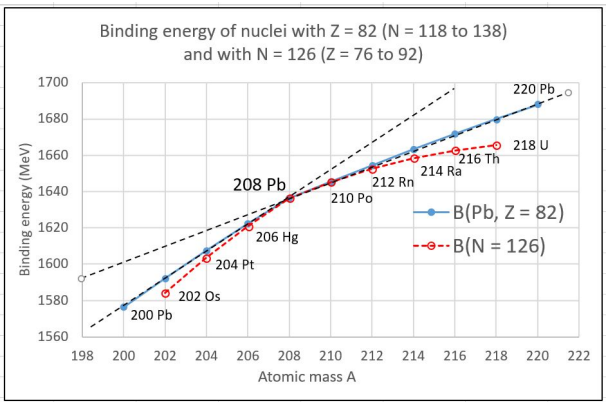


Figure 37: Binding energies of the magic nuclei in the neighborhood of the doubly magic nucleus 208 Pb. The change of the slopes of the curves indicates a change of the geometry of the eigenspaces.

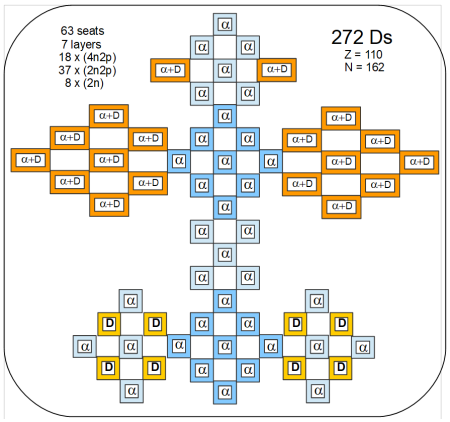


Figure 38: Eigenspace (2D - plot) of the nucleus 272 Ds, it has 63 seats and seven layers. 272 Ds represents the heaviest magic nucleus discovered so far. The symbol α means a (2n2p) - group, D means a dineutron (2n) and α+D represents a (4n2p) - group.

Extrapolations to super heavy nuclei Theoretical calculations yield different predictions for magic numbers of protons and neutrons. Predicted proton magic numbers include Z = 114, 120, 122, 124, 132, and 138. The neutron magic numbers N = 172, 184, 198, 228, 238, and 258 have been calculated using different models ([12]). Other calculations predict ^{270}Hs (Z = 108, N = 162) to be a doubly magic *deformed* nucleus ([10]). The term 'deformed' means non-spherically (prolate) in contrast to the 'spherical' magic numbers 2 up to 126.

The seat concept proposed in this article leads to identically magic numbers for protons and neutrons, also in the region of superheavy nuclei. Only five nuclides with the magic number 126 for neutrons could be observed so far: 264 No, 266 Rf, 268 Sg, 270 Hs, and 272 Ds. They belong to the isotonic branch of a hypothetical crossing point at N = Z = 126 (see Figs. 10 and 12).

Although some theoretical predictions do not prefer Z = 126 as a possible magic number ([15]), we ask for the seat geometry of the magic number 126, valid for neutrons and protons.

We assume, that M_{mag} = 126 is a magic number also for protons, in analogy to M_{mag} = 82 and the other smaller magic numbers. This assumption appears naturally, because the skeleton masses of proton and neutron models are identically within the structure of nuclei (see the figures in the appendix of [14].

Selected nuclides with Z = 126 could possibly have somewhat longer half lifes. We estimate the corresponding neutron numbers by extrapolation between M_{mag} = 50, 82,and 126. We consider the distance between the stable nuclei in an isotopic branch and nuclei at the crossing point (N = Z). The distance is

D = 10 at 120 Sn (stable), the crossing point is at 100 Sn with N = Z = 50;
D = 20 at 204 Pb (stable), the crossing point is at 164 Pb with N = Z = 82;

D = 30 at 312 [126] (probably longer lived),

the crossing point is at N = Z = 126.

The predicted 'longer lived' nuclide has N = 186 neutrons. The region of relative stability could reach from N = 162 to198. This corresponds to A = 288 to 324.

This method of extrapolation is derived from the fact, that the (np) - (np) repulsion can be shielded by dineutrons. The groups (2n2p), (4n2p), and (6n2p) become stepwise 'forcefree' groups. The same steps of increasing numbers D of shielding dineutrons are applied in the extrapolation.

Other methods of extrapolation resulted in the prediction of the nucleus 342 [126] as the center of a 'shallow', see [18].

However, we have no reliable estimation of the life times of the nuclides with Z = 126, and therefore the given estimations do not provide a clear prediction of a 'shallow' or even an 'island of stability'.

The magic number $\mathbf{M}_{mag} = 162$ Although nuclei with the atomic number Z = 162 probably do not exist, we discuss a hypothetic crossing point at Z = N = 162 and a hypothetic isotopic branch with Z = 162 besides the observed isotonic branch with N = 162. A representation of this geometry with 81 seats and eight levels is depicted in Fig. 39.

We remark, that the geometry shown in Fig. 39 is not a simple completion of the seven-wing geometry of the magic number 126. If we would add a wing (eight seats) to the 63 seat geometry of the magic number 126, we would obtain a 71 seat geometry belonging to the (non-magic) number 142. However, there is no sign of a magic number 142. The magic number 162 which can be identified by the 'energy gap' method leads to somewhat extended links between the six wings and the two platforms.

The realization of the seat geometry with 81 seats can be expected at $Z \geq 126$.

Fig. 40 shows the 2D plot of an eigenspace of the nucleus 288[126], the lightest nuclide which is forced to realize this big conformation with 81 seats in eight layers.

Estimation of the size of the models of heavy magic nuclei The root-mean-square charge radii (rms) and the diameters of some heavy nuclei are given in.Table 2 (see [11]).

Table 3: Charge radii of heavy nuclei

nuclide	charge radius (fm)	diam. (fm)
112 Sn	4,5948	9,1896
202 Pb	5,4705	10,941
248 Cm	5,8687	11,7374

We compare the measured charge diameters with the extension of a wing in eigenspaces of nuclear models (see Figs. 27, 36). The figures show the 2D-plots

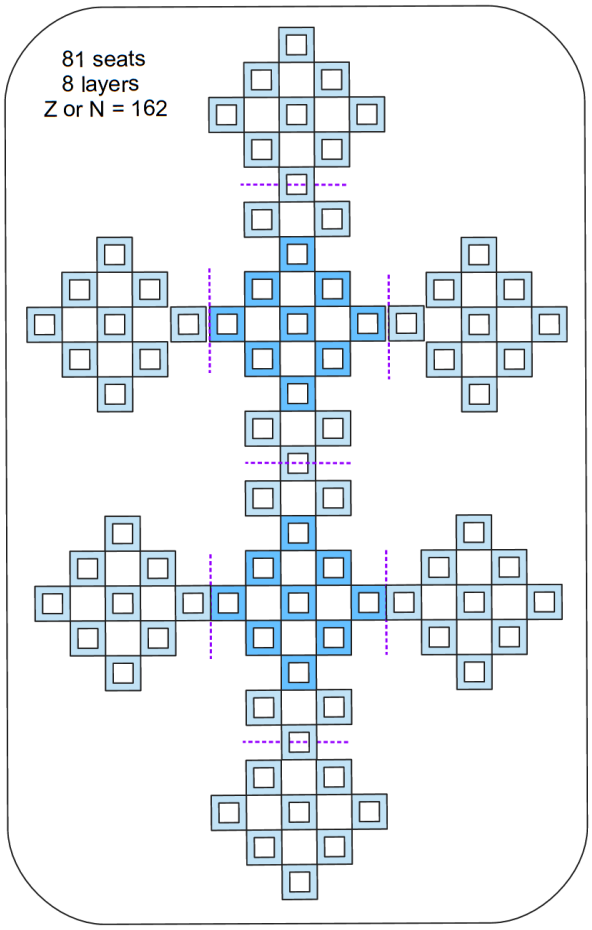


Figure 39: Seat configuration with 81 seats in 8 layers. The observed nuclei with N = 162 do not have this configuration, because the atomic number is Z $<$ 126.

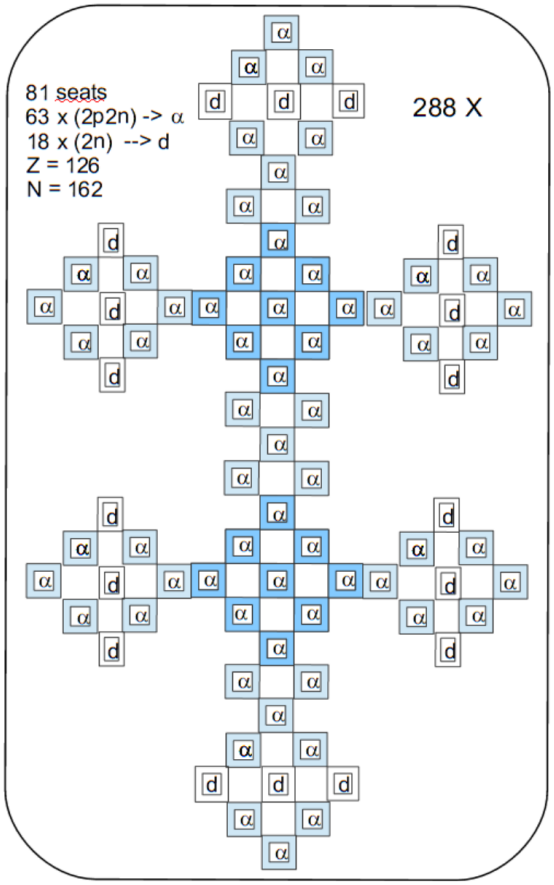


Figure 40: Eigenspace of a hypothetically doubly magic nucleus with (Z, N) = (126, 162) as 2D - plot. It has 81 seats in eight layers. The symbol α means a (2n2p) - group, d means a dineutron (2n).

of the eigenspaces, 'flat' 2D - plots of the real eigenspaces. Besides such a 'flat' representation we consider in Table 4 also the non-flat variant of a wing (see the column 'overlap'). That means, a row or a column of a wing could overlap with other parts of the wing and therefore become shorter and non-flat. A bar of maximum length in a wing consists of 3 seats, occupied by at least 3 (2n2p) - groups.

Table 4: Estimated sizes of wing-like eigenspaces of heavy nuclei

parton	bar length in a wing (fm)	
	with gap (flat)	overlap
HA (m_8)	12,864	9,648
LA (m_{41})	14,832	11,124

Only the non-flat variant (with overlap) of the wings has a plausible relation to the measured charge-diameters listed in Table 3.

3.3.3 The binding energy of magic nuclei

The magic nuclei have eigenspaces of a special symmetry. We select the most stable ones among these symmetric nuclei in each isotopic or isotonic branch belonging to a magic number. 'Most stable' means such nuclides having the maximum value B/A of the average binding energy per nucleon.

By this way we selected 12 nuclides fom Z = 2 to 102, see Table 5.

The following parameters are included in Table 5:

Z = N(np) represents the atomic number as usual and also the number of (np) - pairs as parts of (2n2p) - groups.

N, the number of neutrons

D = (N - Z)/2, the number of dineutrons

C, the number of contact points beween the seats, taken from the seat configurations. In wing-like geometries, one has at least 12 contact points per layer, 36 at $M_{mag} = 50$ (two wings, 3 layers), 60 at $M_{mag} = 82$ (four wings, 5 layers) and 92 at $M_{mag} = 126$ (five wings, 7 layers, 16 contacts per platform).

S, the number of seats belonging to the corresponding magic number, and

L, the number of layers, indicated in the figures of the magic seat configurations.

The configuration with eight layers requires nuclei with Z $\geq$ 126, therefore it is not discovered so far.

(2n2p) - groups and dineutrons represent the structural components of magic nuclei. Combined (4n2p) - groups occupying one seat are possible. The splitting of one (2n2p) group into two (np) groups occupying two seats are excluded in models of magic nuclei, such a situation happens in asymmetric, non - magic nuclei only.

The binding energy of a magic nucleus consists of three components:

$B(np)_2$, the internal energy of an alpha-particle, the special case of a (2n2p) - group,

B_{32}, the binding energy of a contact point between two (2n2p) - groups or a (2n2p) - group and a dineutron, and

Figure 41: Table 5: Binding energy and parameters of selected magic nuclei (magic numbers in red color). B/A and B are empirical values.

nucl.	Z	N	D	C	S	L	B/A (MeV)	B (MeV)
	cross - like geometry							
4 He	**2**	**2**	0	0	1	1	7,0739	28,2957
16 O	**8**	**8**	0	4	4	1	7,9762	127,6193
38 Ar	18	**20**	1	16	10	2	8,6143	327,3426
46 Ca	**20**	26	3	16	10	2	8,6690	398,7730
52 Cr	24	**28**	2	22	14	3	8,7760	456,3514
62 Ni	**28**	34	3	22	14	3	8,7946	545,2623
	wing - like geometry							
88 Sr	38	**50**	6	36	25	3	8,7326	768,4684
116 Sn	**50**	66	8	36	25	3	8,5231	988,6815
136 Xe	54	**82**	14	60	41	5	8,3962	1141,8816
202 Pb	**82**	120	19	60	41	5	7,8822	1592,1943
206 Hg	80	**126**	23	92	63	7	7,8692	1621,0494
264 No	102	**162**	30	92	63	7	7,3710	1945,9440

B(nn), the binding energy of a dineutron. Contact points between two dineutrons do not occur.

The quantities $B(np)_2$ and B_{32} are *averages* over the structural components of the nucleus under consideration. In particular, the binding energy of a dineutron may depend on its position in the eigenspace.

The formula of the binding energy of a magic nucleus $B(Z,N)$ shows the the numbers Z and D of structural components of the nucleus as well as the number C of contact points between them and can be written

$$B(Z,N) = Z * B(np)_2 + D * B(nn) + C * B_{32} \tag{7}$$

This formula combines empirical data and model - assumptions. The empirical B - components for the range of Z = 2 to 102 are given in Fig. 42.

$B(np)_2 = 14.147832$ Mev is constant, independent of Z (see equ. 1).

B_{32} is drawn in the form $B(np)_3 = B(np)_2 + B_{32}$, where B_{32} depends on Z. The average values are

$B_{32,O} = 3.6176\ MeV$ at magic numbers $M_{mag} \leq 50$,

$B_{32,Sn} = 3.30764\ MeV$ at $50 \leq M_{mag} \leq 82$ and

$B_{32,Pb} = 2.45111\ MeV$ at $M_{mag} > 82$.

This moderate decrease of B_{32} with increasing Z is caused by an increasing number D of dineutrons in heavier nuclei. D increases from 6 to 30 for Z = 38 to 102. The dineutrons shield the (np) - (np) - repulsion to a great part.

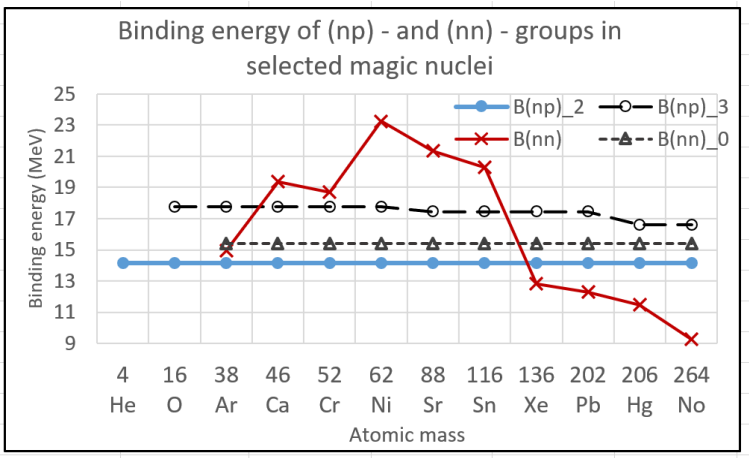


Figure 42: Binding energy of (np) - and (nn) - groups in magic nuclei. The nucleus with the maximum B/A is chosen from each isotopic and isotonic branch of a magic number.

$B(nn)$ differs largely from the 'neutral' value $B(nn)_0 = 15.4\ MeV$. B(nn) reaches values of 19 to 23 MeV between Z = 20 and Z = 50, caused by the dominating (nn) - (np) - attraction. B(nn) decreases in wing-like geometries down to 13 and finally to 9 MeV between Z = 54 and Z = 102. The reason of the decrease is the higher effect of (nn) - (nn) - repulsion compared to the (nn) - (np) - attraction at Z > 50.

We give two numerical examples for equation 7. First, we consider the binding energy of 62 Ni, the nuclide with the highest value of B/A. With $C = 22$ as the number of contact points (see Fig. 17) and $D = 3$ as the number of dineutrons one obtains the experimental value:

$B(62\ Ni) = 28 * B(np)_2 + 3 * 23.24 + 22 * B_{32,O} = 545.2623\ MeV$

The second example is the binding energy of 202 Pb, one of the heaviest stable nuclei:

$B(202\ Pb) = 82 * B(np)_2 + 19 * 12.295 + 60 * B_{32,Sn} = 1592.1943\ MeV.$

We find $C = 60$ as the number of contact points in the configuration with 41 seats and five layers. We count 12 points per layer, see Fig. 42.

$D = (N - Z)/2 = (120 - 82)/2 = 19$ is the number of dineutrons in a 202 Pb - nucleus.

The formulas for $B(62\ Ni)$ and $B(202\ Pb)$ demonstrate the applicability of the model of nuclear coupling proposed in this article and in [14]. However, the formulas do not represent theoretical calculations of the binding energies because the B - factors shown in Fig. 42 have an empirical origin. Formula (7) corresponds a bottom - up method of theoretical interpretation.

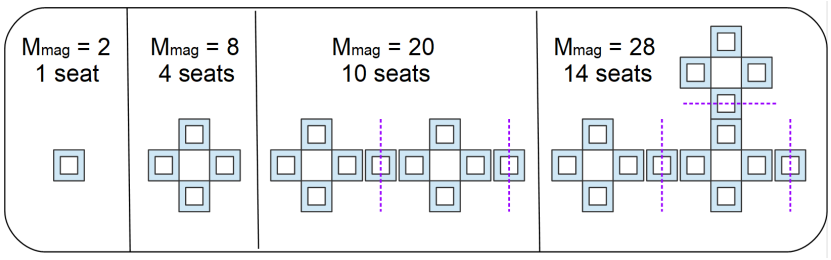


Figure 43: Seat configurations of the magic numbers 2, 8, 20, and 28, showing a cross - like symmetric geometry. The eigenspaces have one to three layers and are drawn levelled as 2D - plots.

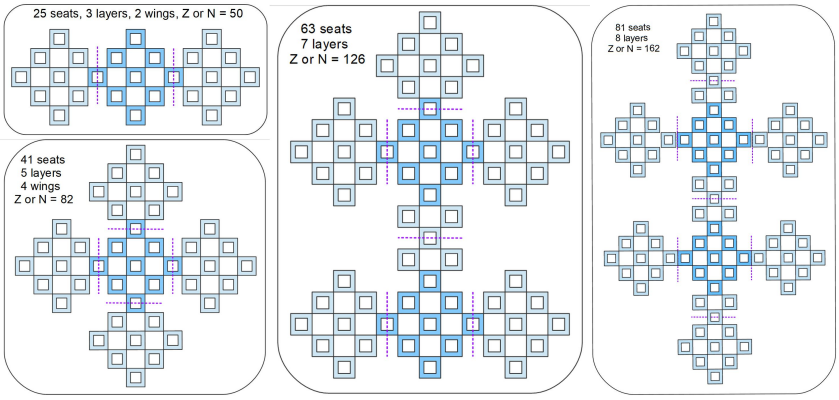


Figure 44: Seat configurations of the magic numbers 50, 82, 126, and 162, showing a wing - like symmetric geometry. The eigenspaces have three to eight layers and are drawn levelled as 2D - plots.

3.4 Overview on magic numbers

The magic numbers have a geometric origin. The two parts of the series of eight numbers - the small and the big magic numbers - are separated by energetic and geometrical, not in the first place by mathematical arguments.

The figures of the seat configurations provide a visual impression of the special symmetry of the geometric representations, see Figs. 43 and 44. Table 6 contains an overview on the geometry. The geometric properties of the eigenspaces are identically, independent of the cases $Z = M_{mag}$ or $N = M_{mag}$.

Table 6: Seat configurations belonging to the magic number M_{mag}

M_{mag}	Number of seats	Layers	Geometry
2	1	1	point (1 seat)
8	4	1	2 bars (1 cross)
20	2 x 4 + 2 = 10	2	2 x 2 bars, links (2 crosses)
28	3 x 4 + 2 = 14	3	3 x 2 bars, links (3 crosses)
50	2 x 8 + 9 = 25	3	2 wings, 1 platform
82	4 x 8 + 9 = 41	5	4 wings, 1 platform
126	5 x 8 + 2 x 9 + 5 = 63	7	5 wings, 2 platforms, links
162	6 x 8 + 2 x 9 + 15 = 81	8	6 wings, 2 platforms, links

4 Conclusions

The definition of 'magicity' of nuclei is based on a single criterion, the gap in the separation energy of a dineutron. Using this criterion, the four small magic numbers 2, 8, 20, and 28, as well as the four large magic numbers 50, 82, 126, and 162 were identified. 'Magicity' of a nucleus requires an even neutron number N and an even proton number Z, the spin - parity signature is always 0^+. Thus only bosonic, spin-compensated groups of nucleons appear as components of magic nuclei, such as (np)-, (nn)- and (pp) - groups with group - spin zero. In particular the (2n2p) – group within a nucleus is of importance. The (2n2p) - group shows an internal compensation of attraction and repulsion between the four nucleons. This property is called 'force-compensation' and enables a constant contact energy between two (2n2p) – groups within a nucleus, independent of the total number of nucleons, within a wide range of Z values.

The common eigenspace of all nucleons in a magic nucleus can be levelled and transformed into a plane, drawn as a 2D - plot. A seat - concept was developed to provide a lattice - like geometric framework of placeholders for different nuclear groups. The same unified seat configuration can be used to construct structural schemata of the magic nuclei. A unified seat configuration is applicable for nuclei of the isotopic branch (where Z equals a magic number M_{mag}) and of the isotonic branch (where N equals a magic number M_{mag}).

Each magic number is characterized by a special symmetry of the seat configuration. The symmetry is point - like for $M_{mag} = 2$, and cross - like for the light magic numbers 8, 20 and 28, where the structure is folded in one, two or three layers. The big magic numbers have a wing – like geometry with several symmetric 'wings' in three to eight layers. The special symmetry of magic eigenspaces provides energetically unfavorable conditions for the binding of a next bosonic component. This situation corresponds to a filled shell in the nuclear shell model, where the next nucleon starts a new shell or subshell with a lower binding energy.

The size of the magic eigenspaces of nuclei was estimated and compared with the measured charge radii of the nuclei in spacetime. A plausible relation between the extension in the eigenspace and in spacetime exists for the alpha-

particle. For more complex nuclei, additional assumptions on the geometry of the eigenspace in three dimensions are necessary.

The extrapolation of the geometry of eigenspaces into the region of superheavy nuclei up to Z = 126 is discussed.

The seat concept leads to magic numbers identically for protons and neutrons, also in the region of superheavy nuclei. In contrast to different theoretical calculations, the geometrical seat concept predicts a 'magic' isotopic branch of nuclei with Z = 126. Some nuclei within this branch could probably have a longer lifetime than its neighbors. This would be a similar effect as within the isotonic branch belonging to $M_{mag} = 126$, for instance concerning the nucleus ^{270}Hs (Z = 108, N = 162).

5 Appendix: Energy factors of circulating charges as function of $z = \alpha/\pi$

$m_Q = 3.252\,848\,865\ MeV$ mass quantum (energy - equivalent)
$z = \alpha/\pi = 2.322\,819\,465 \times 10^{-3}$
$b_k = 1 + kz$; example $b_4 = 1.009\,291\,278$
$d_k = 1 + kz + (kz)^2$
$a_k = 1 + kz + (kz)^2 + (kz)^3$
$w_k = 1 + kz + (kz)^2 + (kz)^3 + ... = \frac{1}{1-kz}$; example $w_4 = 1.009\,378\,415\,3$

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